

実空間密度汎関数法による ナノ物質の第一原理 シミュレーション

実空間密度汎関数法コード - *RSDFT* -

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Material Science
- 筑波大 (櫻井研) 二村保徳 (櫻井-杉浦の固有値解法)
 - 東大 (押山研) 内田和之 (二層グラフェン)
 - 東大 (押山研) 澤田啓介 (SiCステップ表面)



ACM Gordon Bell Prize Peak Performance

**Yukihiro Hasegawa, Junichi Iwata, Miwako Tsuji,
Daisuke Takahashi, Atsushi Oshiyama,
Kazuo Minami, Taisuke Boku, Fumiyoshi Shoji,
Atsuya Uno, Motoyoshi Kurokawa, Hikaru Inoue,
Ikuo Miyoshi, Mitsuo Yokokawa**

*First-Principles Calculation of Electronic States of a
Silicon Nanowire with 100,000 Atoms on the K Computer*



Scott Lathrop
Scott Lathrop
SC11 Conference Chair

Thom H. Dunning, Jr.
Thom H. Dunning, Jr.
Gordon Bell Chair



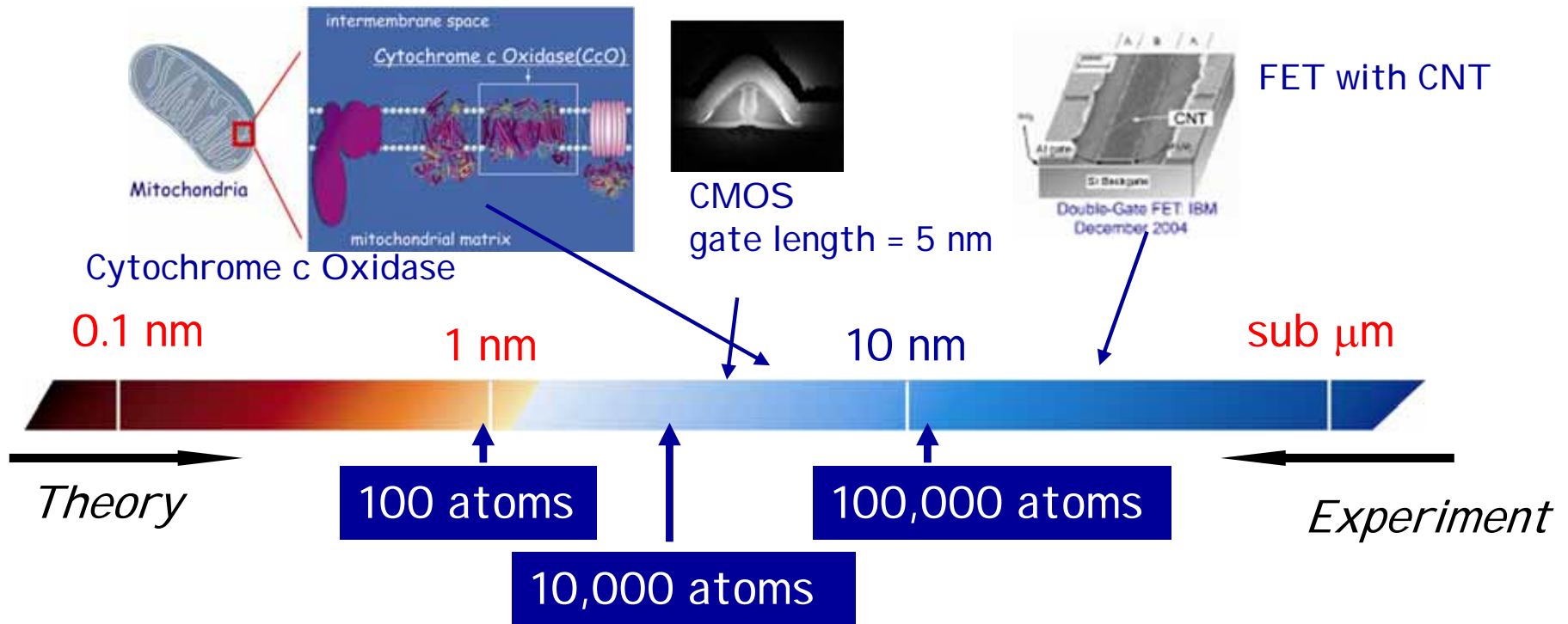
内容

- 第一原理電子状態計算
 - 密度汎関数法
 - RSDFT
- Siナノワイヤ
- 櫻井-杉浦法のバンド構造計算への応用（筑波大 二村保徳）
- 擦じれた二層グラフェン（東大 内田和之）
- SiCステップ表面（東大 澤田啓介）
- RSDFT-CPMDの実装（東大 小泉健一）

FIRST-PRINCIPLES CALCULATIONS

Density
Functional
Theory

LARGE-SCALE FIRST-PRINCIPLES CALCULATIONS IN NANO WORLD



Large-scale DFT calculations and experiments meet together in Nano World !

Challenge: 10,000 ~ 100,000-atom calculations overcoming N^3 scaling to reveal nano-scale world!

DENSITY FUNCTIONAL THEORY

Energy functional

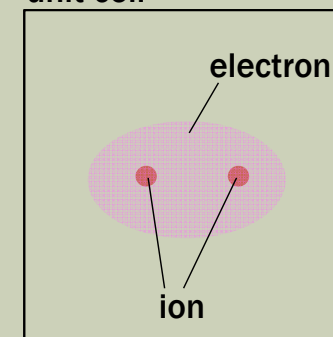
$$E[\{\phi_i\}] = \sum_{i=1}^N \int d\mathbf{r} \phi_i^*(\mathbf{r}) \left(-\frac{1}{2} \nabla^2 \phi_i(\mathbf{r}) \right) \quad \text{運動エネルギー}$$

$$+ \int d\mathbf{r} \rho(\mathbf{r}) v_{ion}(\mathbf{r}) \quad \text{電子-イオン相互作用}$$

$$+ \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \quad \text{電子間クーロン相互作用}$$

$$+ E_{xc}[\rho] \quad \text{その他量子力学的効果 (交換相関効果)}$$

unit cell



Electron density

$$\rho(\mathbf{r}) = \sum_{i=1}^N |\phi_i(\mathbf{r})|^2$$

P. Hohenberg and W. Kohn, Phys. Rev. 136 (1964) B864.

W. Kohn and L. J. Sham, Phys. Rev. 140 (1965) A1133.

KOHN-SHAM EQUATION

汎関数最小化

$$\frac{\delta E[\phi]}{\delta \phi_i^*(\mathbf{r})} = 0 \quad \left(\begin{array}{l} \text{constraint} \\ \int d\mathbf{r} \phi_i^*(\mathbf{r}) \phi_j(\mathbf{r}) = \delta_{ij} \end{array} \right)$$



系の最安定状態 (基底状態)

変分方程式 : **Kohn-Sham方程式**

W. Kohn and L. J. Sham, Phys. Rev. 140 (1965) A1133.

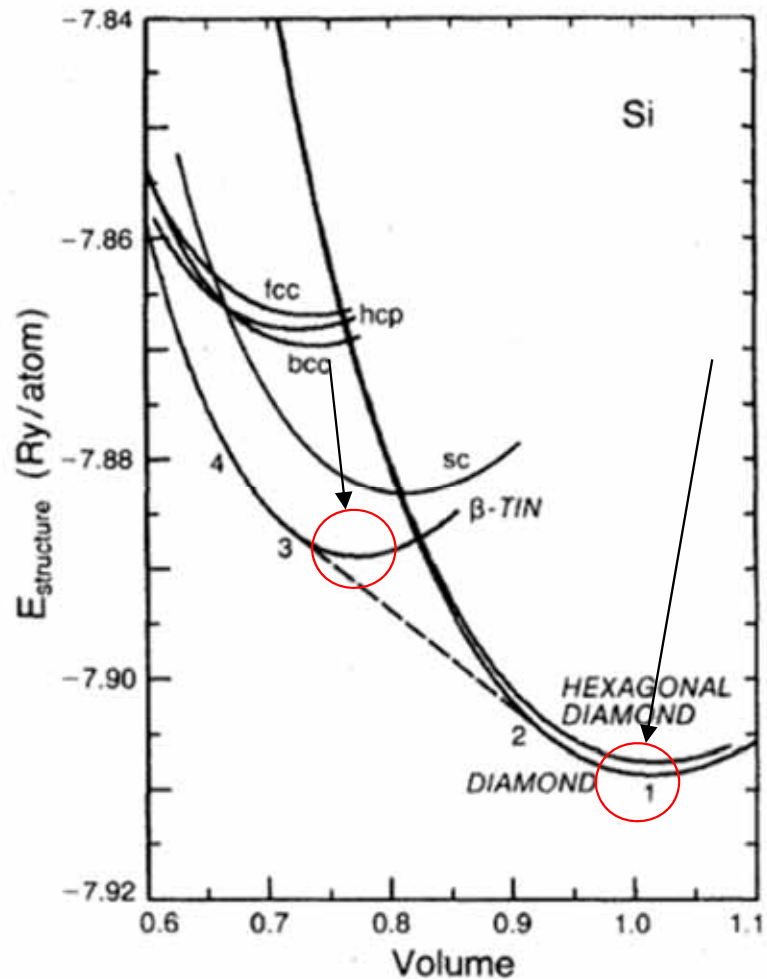
$$\left(-\frac{1}{2} \nabla^2 + v_{KS}(\mathbf{r}) \right) \phi_i(\mathbf{r}) = \varepsilon_i \phi_i(\mathbf{r})$$

電子密度 $\rho(\mathbf{r}) = \sum_{i=1}^N |\phi_i(\mathbf{r})|^2$

$$v_{KS}(\mathbf{r}) = \int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} + \frac{\delta E_{XC}}{\delta \rho(\mathbf{r})} + v_{ion}(\mathbf{r})$$

ポテンシャル

密度汎関数法の威力



単純な近似で

$$E_{xc}[\rho] = E_x[\rho] + E_c[\rho]$$

局所密度近似(LDA)による交換汎関数

$$E_x[\rho] = -C_x \int d\mathbf{r} \rho^{5/3}(\mathbf{r})$$

$$v_x[\rho](\mathbf{r}) = \frac{\delta E_x[\rho]}{\delta \rho(\mathbf{r})} = -\frac{5}{3} C_x \rho^{1/3}(\mathbf{r})$$

高い定量性

Si (ダイヤモンド構造)

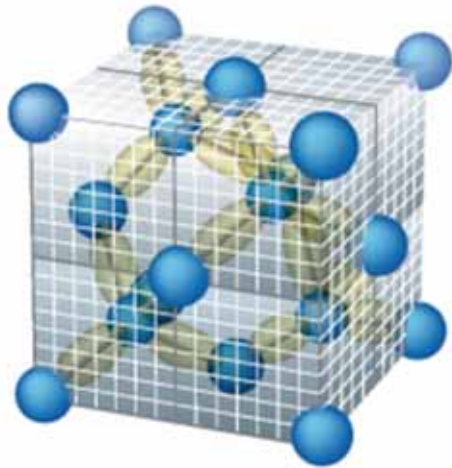
	DFT計算	実験値
格子定数()	5.37	5.41
体積弾性率(Mb)	0.977	0.988

DFT CALCULATION IN REAL-SPACE GRID METHOD

RSDFT

REAL-SPACE FINITE-DIFFERENCE PSEUDOPOTENTIAL METHOD

FFT free



$$\left(-\frac{1}{2} \nabla^2 + \hat{v}_{ion} + \int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} + \frac{\delta E_{XC}[\rho]}{\delta \rho(\mathbf{r})} \right) \phi_j(\mathbf{r}) = \varepsilon_j \phi_j(\mathbf{r})$$

→ **Kohn-Sham equation is solved in discretized space**

J. R. Chelikowsky *et al.*, Phys. Rev. B50, 11355 (1994).

J.-I. Iwata *et al.*, J. Comp. Phys. 229, 2339 (2010).

➤ Derivatives → **(higher-order) finite difference** $\frac{\partial^2}{\partial x^2} \phi(\mathbf{r}) \approx \sum_{m=-6}^6 C_m \phi(x+m\Delta, y, z)$

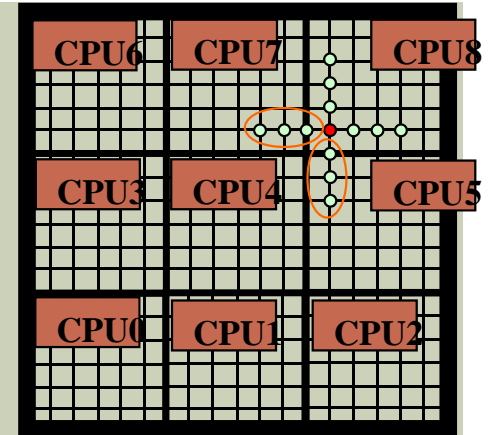
➤ Integrals → **summation over grid points** $\int \phi_m^*(\mathbf{r}) \phi_n(\mathbf{r}) d\mathbf{r} \approx \sum_{i=1}^{N_{grid}} \phi_m^*(\mathbf{r}_i) \phi_n(\mathbf{r}_i) \Delta V$

➤ Ionic potentials → **Pseudopotentials** $\hat{v}_{ion} = v_{local}(\mathbf{r}) + \sum_{a,l,m} |\beta_{alm}\rangle \langle \beta_{alm}|$

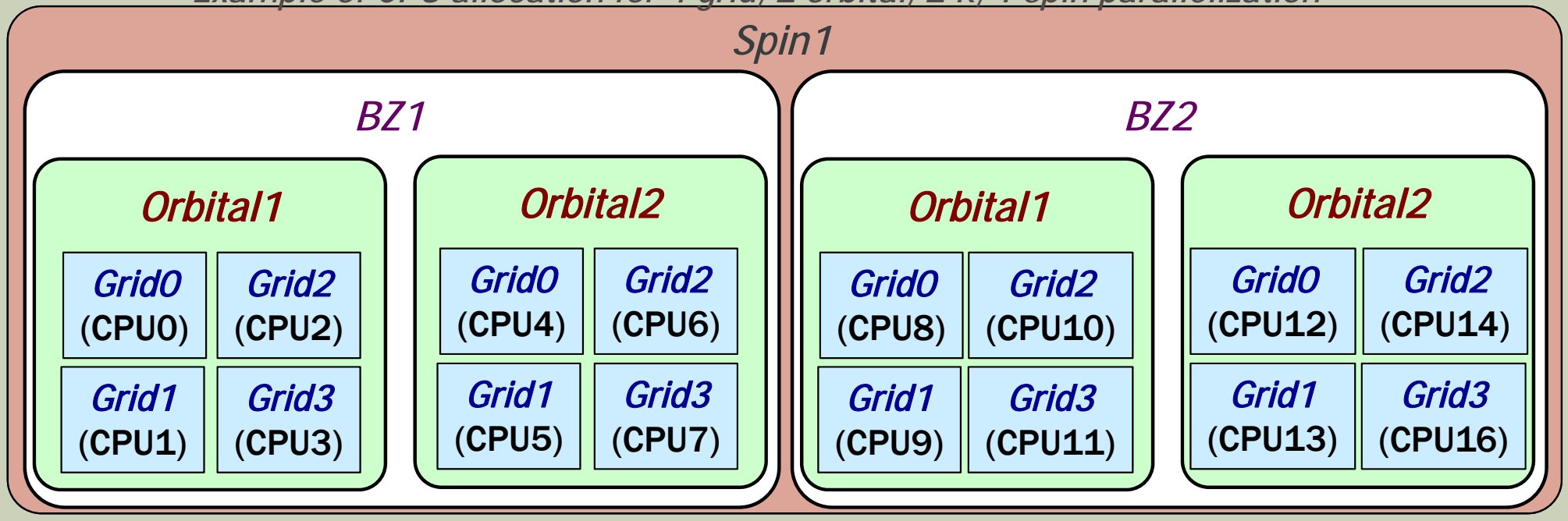
N. Troullier & J. L. Martins
Phys. Rev. B 34, 1993 (1991)

GRID, BAND, k, SPIN PARALLELIZATION

- ◆ MPI (Message-Passing Interface) library
 - MPI_ISEND, MPI_IRECV → **finite-difference** calc.
 - MPI_ALLREDUCE → **global summation**
- ◆ OpenMP
 - Further grid parallelization (within each CPU)
 - is performed by thread parallelization



Example of CPU allocation for 4-grid, 2-orbital, 2-k, 1-spin parallelization



計算上の問題点

SiNW110 (20nm diameter) ML=660 x 660 x 12
6309 atoms MB=13440
3600 nodes 4 sample k points

(second)

	SCF	DIAG	GS	CG
Env_base_1.2.0-07				
Env_base_1.2.0-09				
Env_base_1.2.0-09 (with mca options)				

Details of DIAG routine

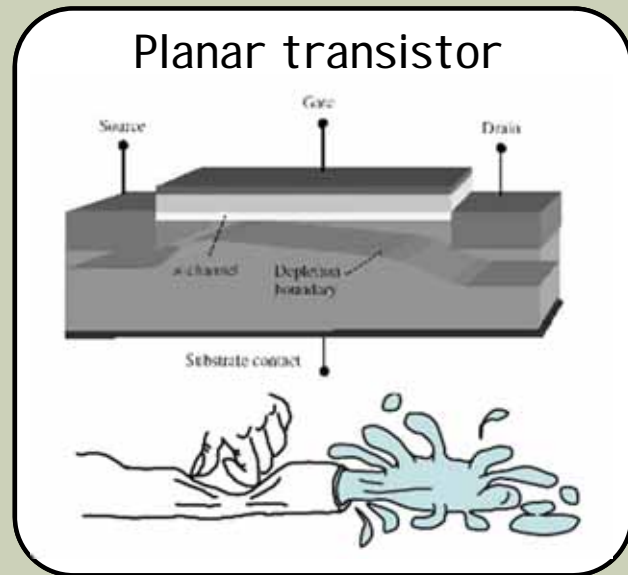
(second)

	DIAG	mate	hpsi	pdsyevd	rotv
Env_base_1.2.0-07					
Env_base_1.2.0-09					
Env_base_1.2.0-09 (with mca options)					

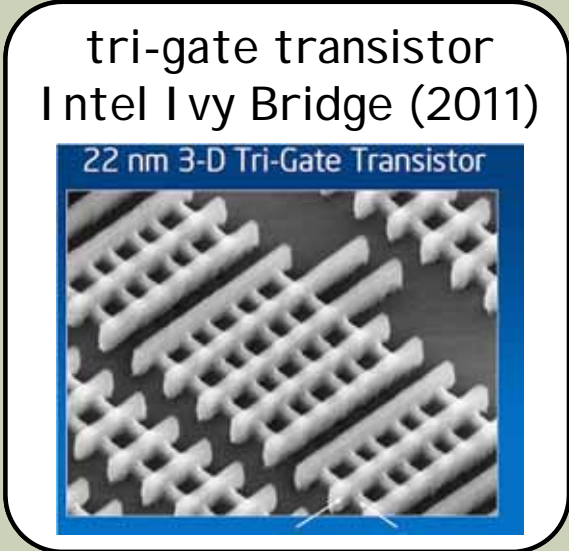
We found MPI_BCAST is the downfall

**NATORI'S COMPACT
MODEL FOR BALLISTIC
Si NANOWIRE MOSFET**

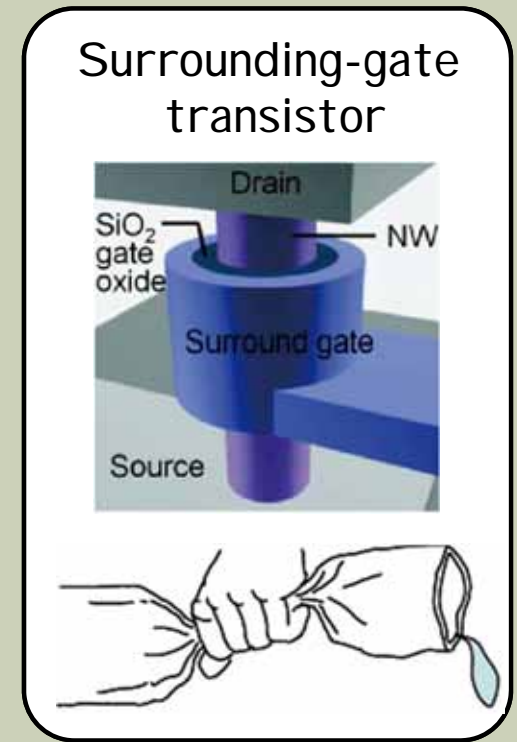
NEW TRANSISTOR STRUCTURES - SUPPRESSION OF OFF-LEAK CURRENT -



Power consumption by off-leak current substantially increases as scaling down of planer FET



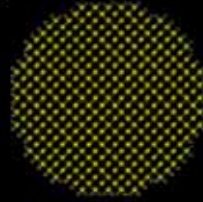
Gate controllability
→ suppress leaks at off state
→ reduce power consumption



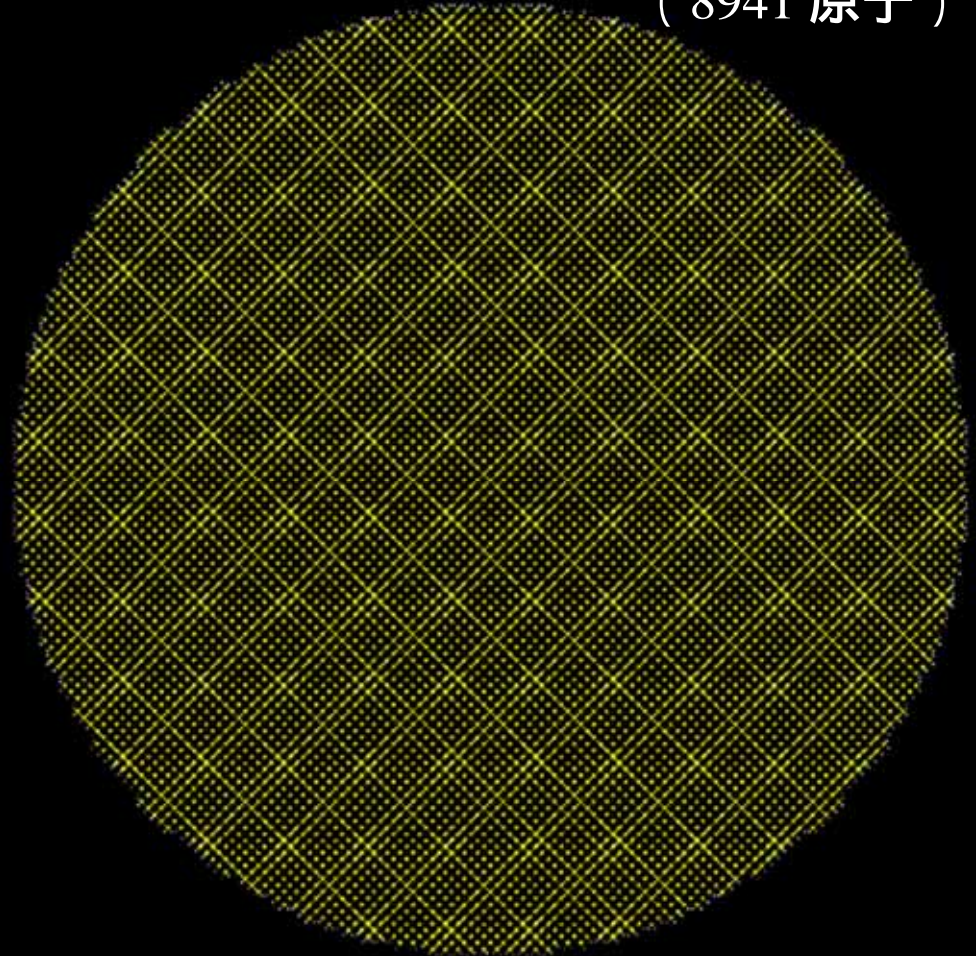
Silicon Nanowire is the most promising channel material for SGFET

シリコンナノワイヤの原子構造 (100)断面

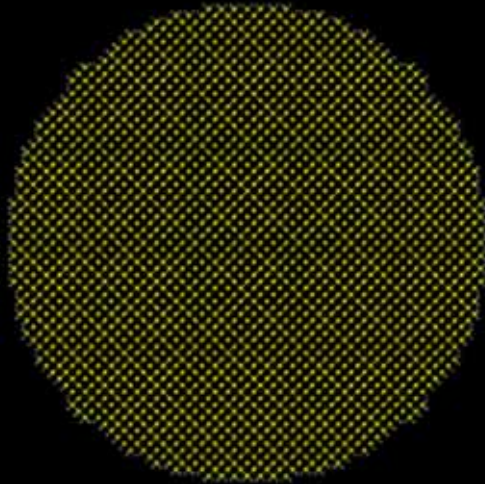
断面直径 4 nm
(425 原子)



断面直径 20 nm
(8941 原子)

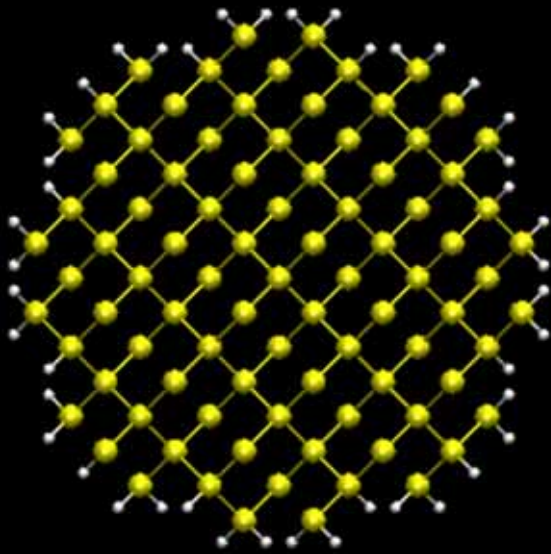


断面直径 10 nm
(2341 原子)

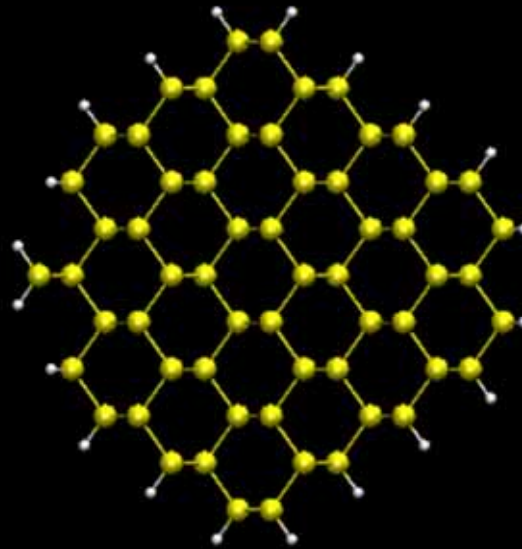


断面サイズ < 20 nm 辺りが
実用になると目されている

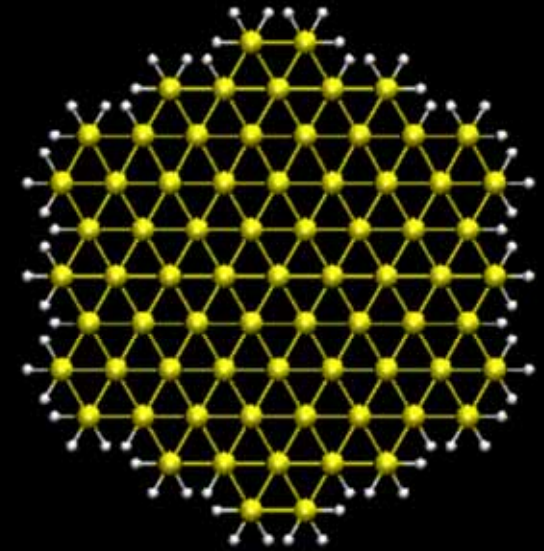
CROSS SECTIONAL VIEWS OF SiNWs



$D=1.96\text{nm}$
[001]



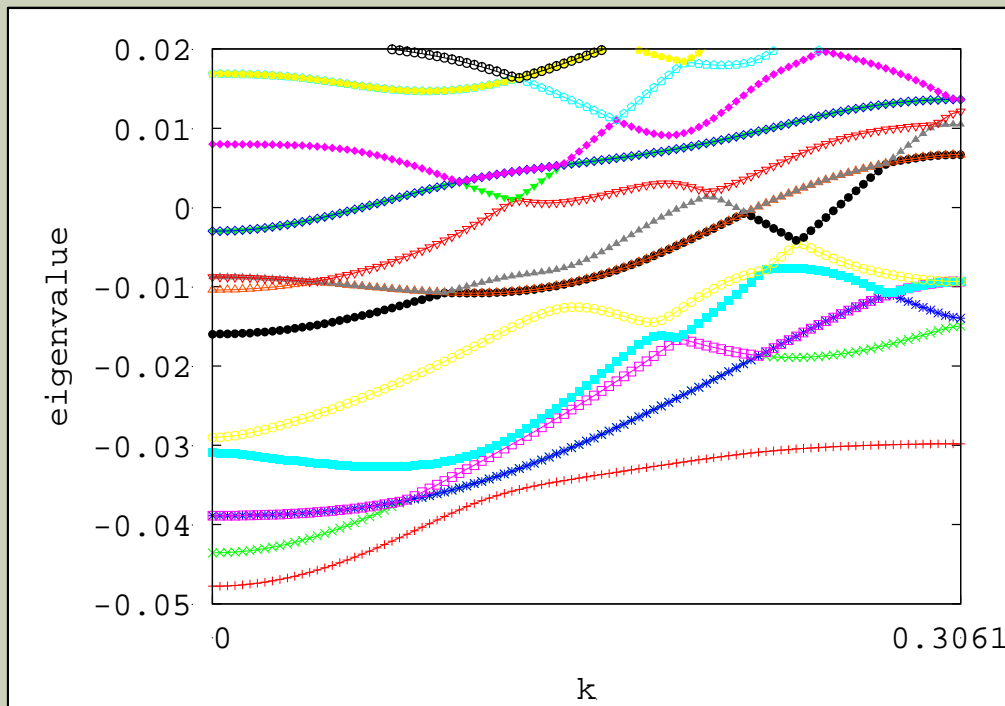
$D=1.94\text{nm}$
[011]



$D=1.93\text{nm}$
[111]

BAND STRUCTURE OF SiNW

$$\left(-\frac{1}{2} \nabla^2 + v_{SCF}(\mathbf{r}) + i\mathbf{k} \cdot \nabla + \frac{k^2}{2} \right) \psi_{nk}(\mathbf{r}) = \varepsilon_{nk} \psi_{nk}(\mathbf{r})$$

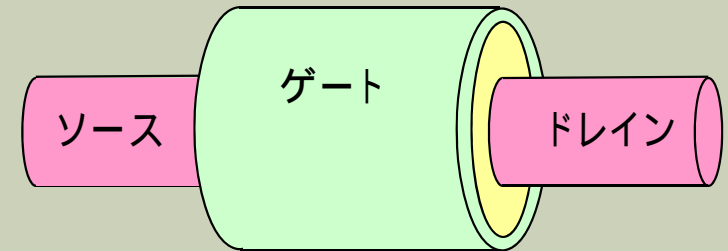
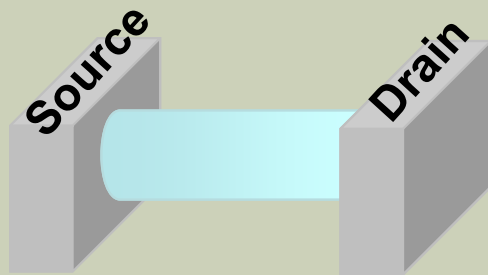


n番目の状態にある電子の速度

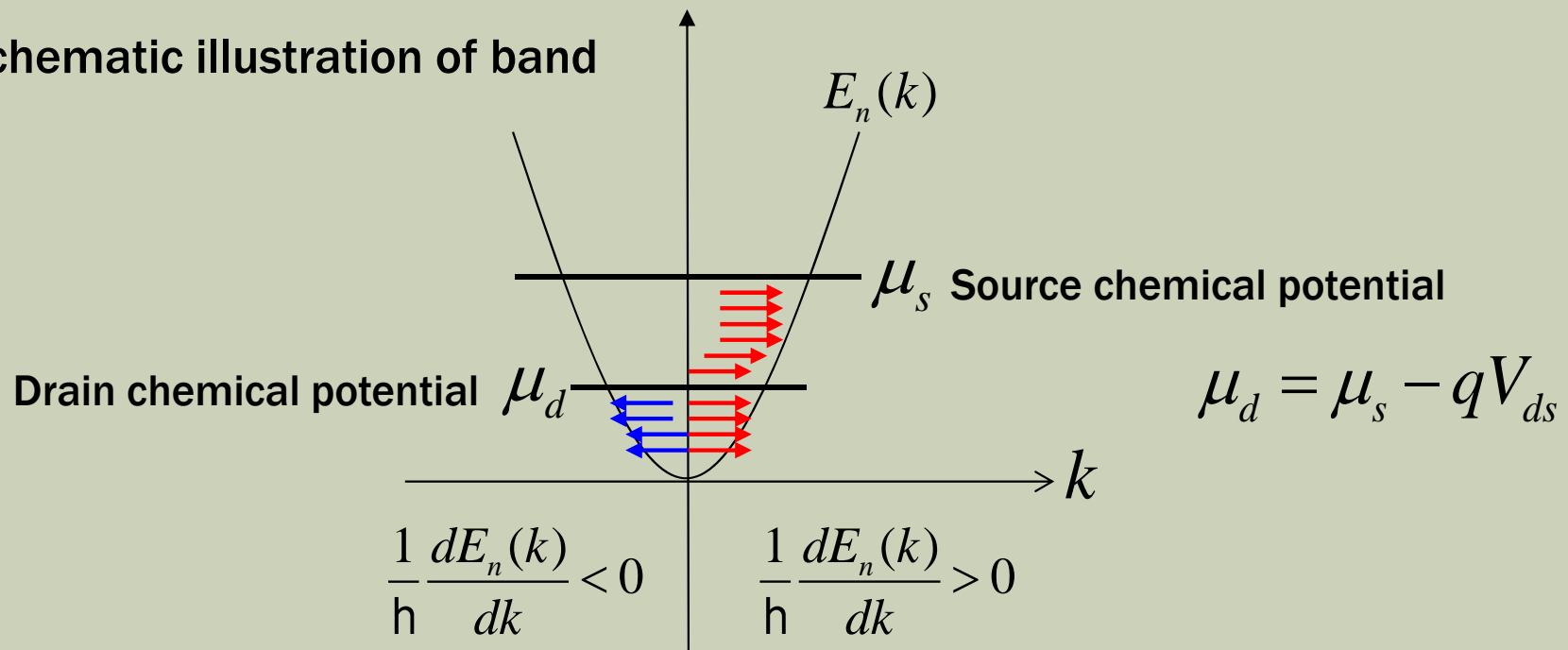
$$\frac{d\varepsilon_n(k)}{dk} = \langle \psi_{nk} | -i\nabla | \psi_{nk} \rangle$$

Derivative of the band energy was calculated by r.h.s. of the following formula

コンパクトモデルによる 電流-電圧特性の評価

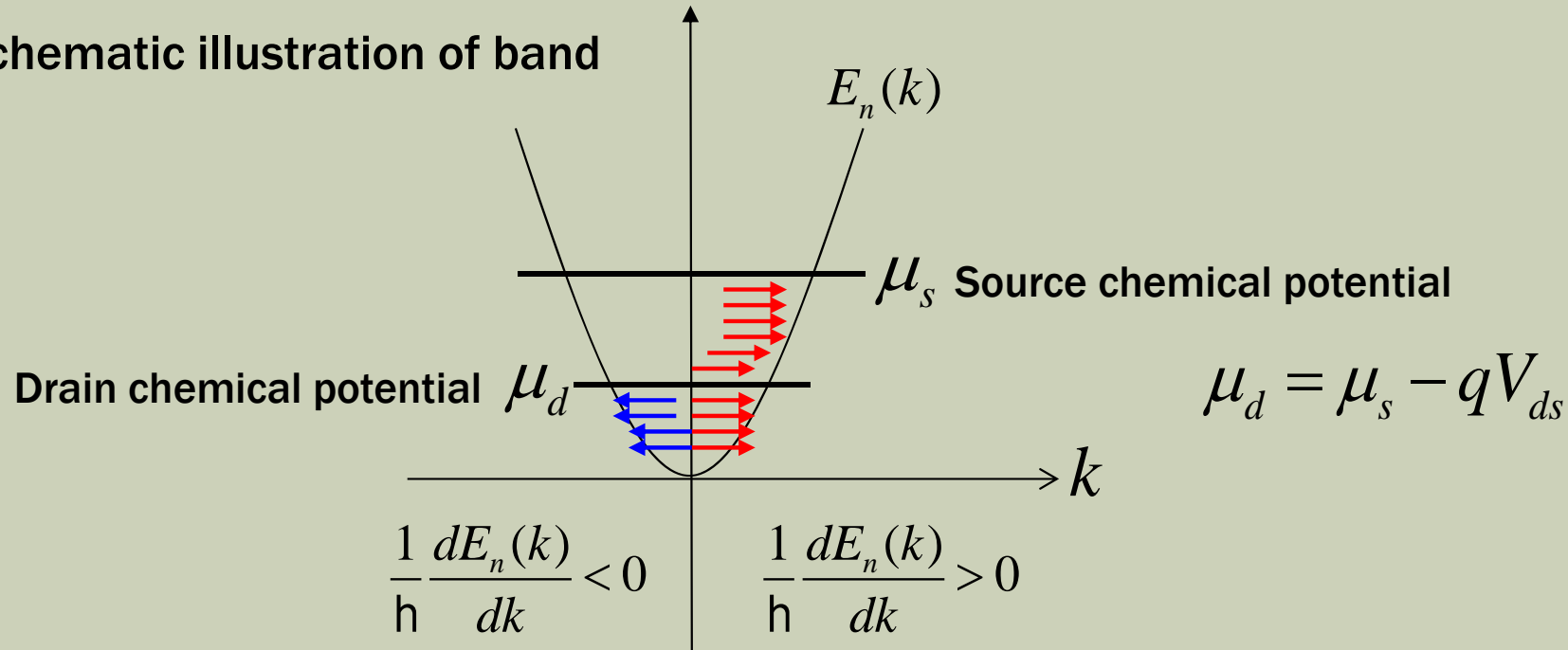


Schematic illustration of band



NUMBER OF CARRIERS IN THE CHANNEL

Schematic illustration of band

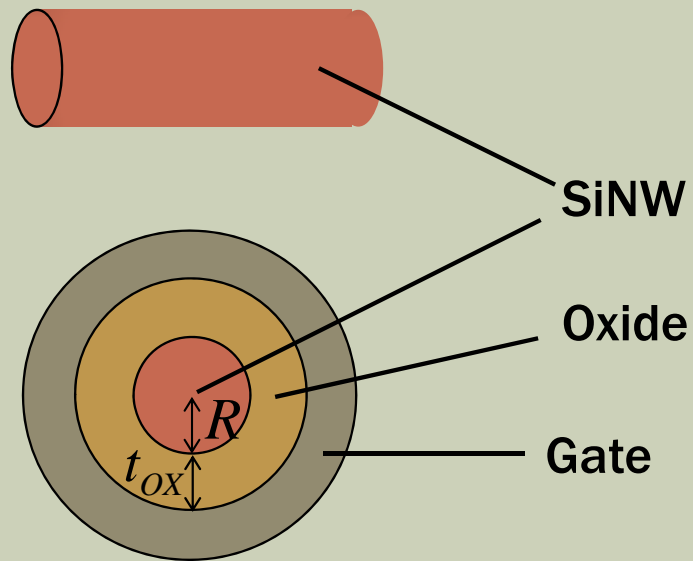


$$N_e^{Fore} = \sum_n 2 \int_{\frac{dE_n(k)}{dk} > 0} \frac{dk}{2\pi} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_s}{kT}\right)}$$

$$N_e^{Back} = \sum_n 2 \int_{\frac{dE_n(k)}{dk} < 0} \frac{dk}{2\pi} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_d}{kT}\right)}$$

$$N_e^{Channel} = N_e^{Fore} + N_e^{Back}$$

NUMBER OF CARRIERS IN THE CHANNEL



$$Q_e^{Channel} = q(N_e^{Fore} + N_e^{Back})$$

$$C_g = \frac{\epsilon_{OX}}{2 \ln\left(\frac{R + t_{OX}}{R}\right)}$$

$$Q_e^{Channel} = C_g \phi_g$$

$$\phi_g = V_g - V_{th} - \frac{\mu_s - E_{CBM}}{q}$$

CURRENT FORMULA

$$I_d = 2q \sum_n \int_{\frac{dE_n(k)}{dk} > 0} \frac{dk}{2\pi} \frac{1}{h} \frac{dE_n(k)}{dk} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_s}{kT}\right)}$$
$$+ 2q \sum_n \int_{\frac{dE_n(k)}{dk} < 0} \frac{dk}{2\pi} \frac{1}{h} \frac{dE_n(k)}{dk} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_d}{kT}\right)}$$

$$= \frac{q}{\pi h} \sum_n \int dE_n \frac{1}{1 + \exp\left(\frac{E_n - \mu_s}{kT}\right)} - \frac{q}{\pi h} \sum_n \int dE_n \frac{1}{1 + \exp\left(\frac{E_n - \mu_d}{kT}\right)}$$

$$= \frac{q}{\pi h} \sum_n \int dE_n (f(E_n, \mu_s) - f(E_n, \mu_d))$$

Landauer formula

$$I = \frac{q}{\pi h} \sum_n T_n(E) \int dE (f(E, \mu_s) - f(E, \mu_d))$$

CALCULATION PROCEDURE

Perform band calculation

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + v(\mathbf{r}) \right) \psi_{nk}(\mathbf{r}) = E_n(k) \psi_{nk}(\mathbf{r})$$

Get

$$\begin{cases} E_n(k) \\ \frac{1}{\hbar} \frac{dE_n(k)}{dk} = \frac{1}{m} \langle \psi_{nk} | -i\hbar \nabla | \psi_{nk} \rangle \end{cases}$$

Give $V_g - V_{th}$ and V_{ds} as input parameters, and solve the following equation

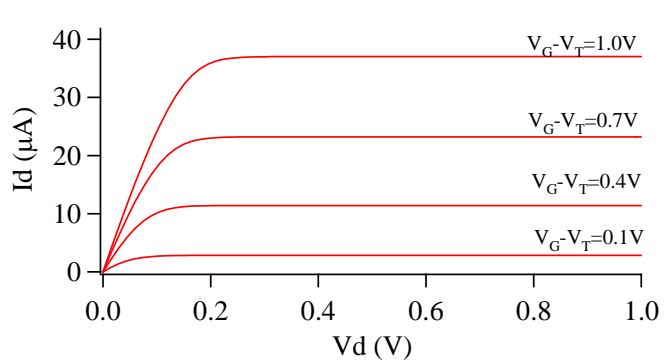
$$\begin{cases} C_g \left(V_g - V_{th} - \frac{\mu_s - E_{CBM}}{q} \right) = Q_e^{Channel} = q \left(N_e^{Fore} + N_e^{Back} \right) \\ N_e^{Fore} = \sum_n 2 \int_{\frac{dE_n(k)}{dk} > 0} \frac{dk}{2\pi} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_s}{kT}\right)} \\ N_e^{Back} = \sum_n 2 \int_{\frac{dE_n(k)}{dk} < 0} \frac{dk}{2\pi} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_d}{kT}\right)} \\ \mu_d = \mu_s - qV_{ds} \end{cases}$$

→ Get μ_s

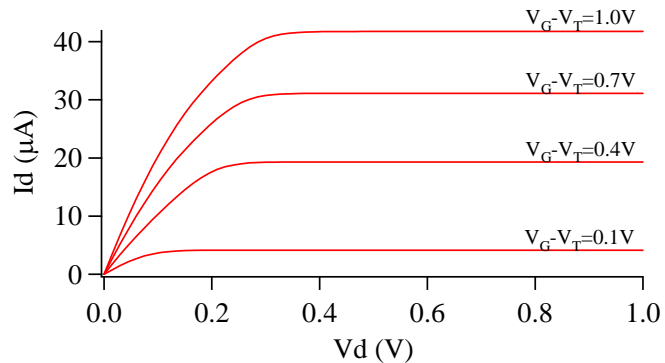
CALCULATION PROCEDURE

Calculate the drain current by the following formula

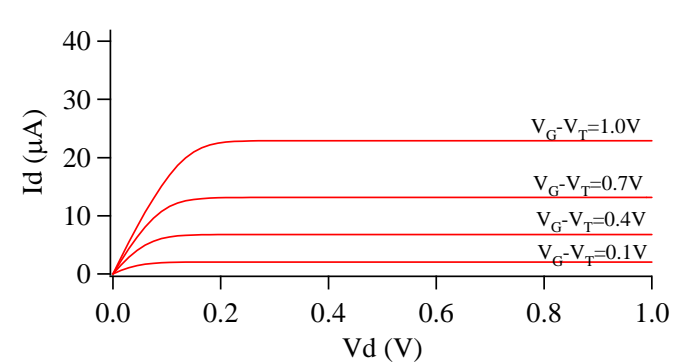
$$I_d = 2q \sum_n \int_{\frac{dE_n(k)}{dk} > 0} \frac{dk}{2\pi} \frac{1}{h} \frac{dE_n(k)}{dk} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_s}{kT}\right)}$$
$$+ 2q \sum_n \int_{\frac{dE_n(k)}{dk} < 0} \frac{dk}{2\pi} \frac{1}{h} \frac{dE_n(k)}{dk} \frac{1}{1 + \exp\left(\frac{E_n(k) - \mu_d}{kT}\right)}$$



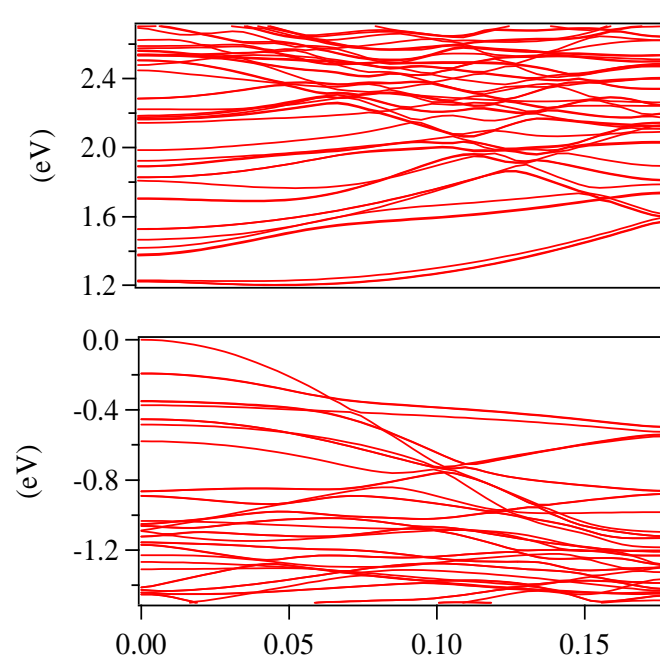
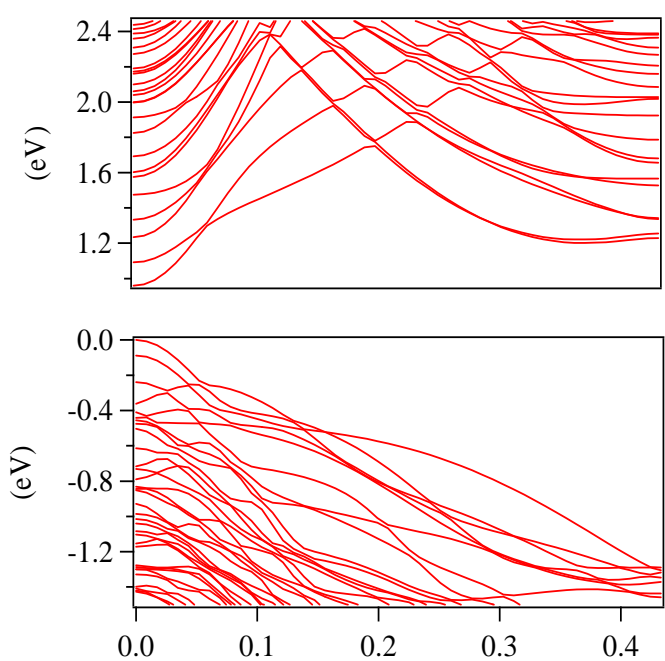
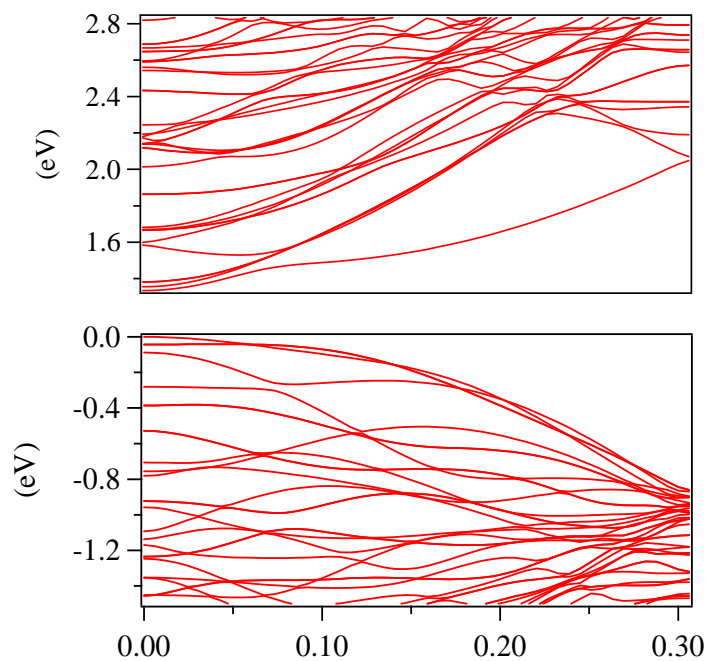
SiNW(100)
 diameter : 2nm
 temperature : 300K

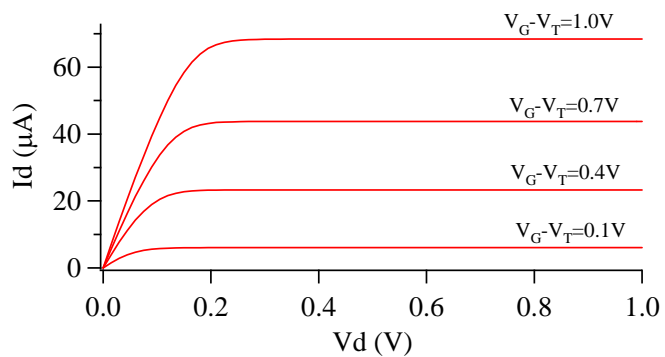


SiNW(110)
 diameter : 2nm
 temperature : 300K

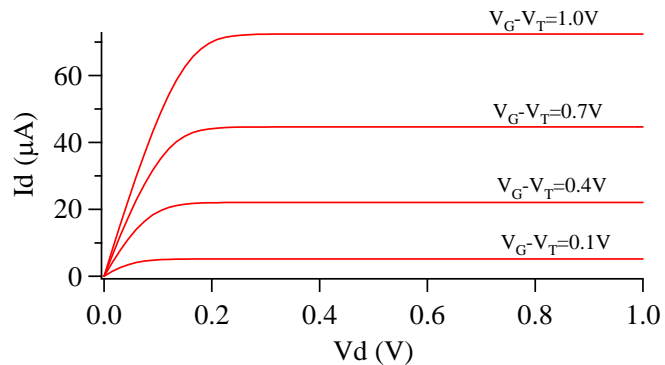


SiNW(111)
 diameter : 2nm
 temperature : 300K

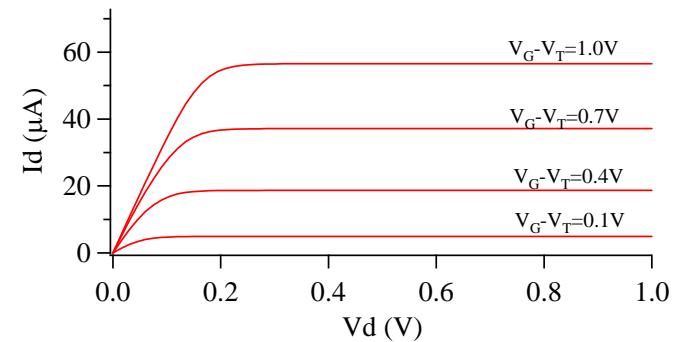




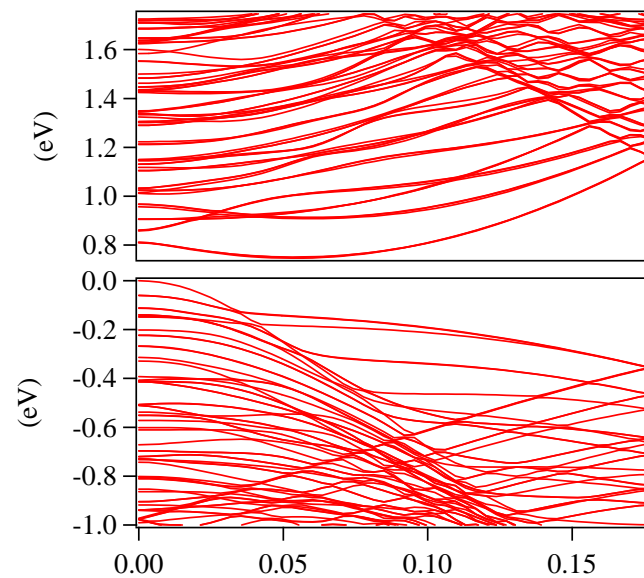
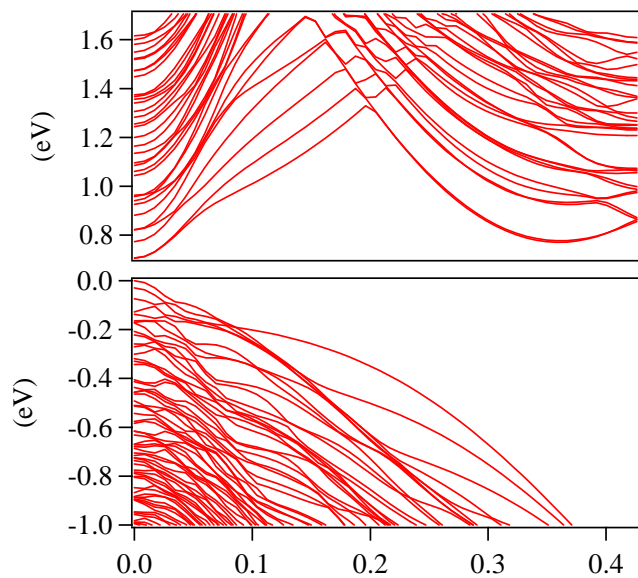
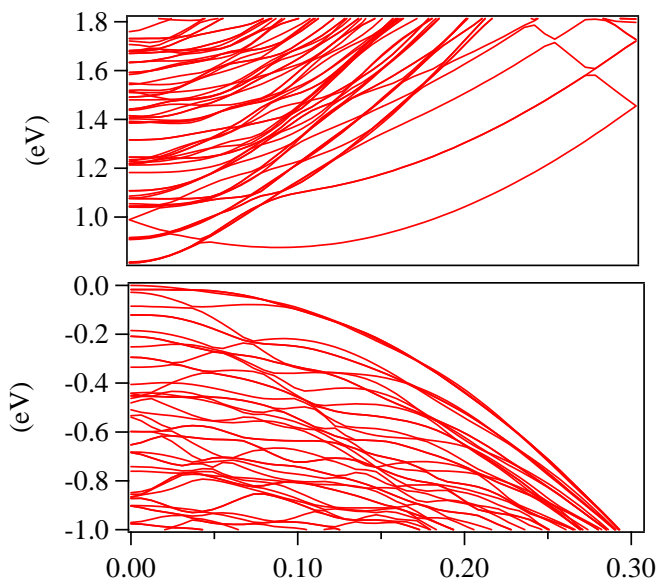
SiNW(100)
 diameter : 4nm
 temperature : 300K



SiNW(110)
 diameter : 4nm
 temperature : 300K

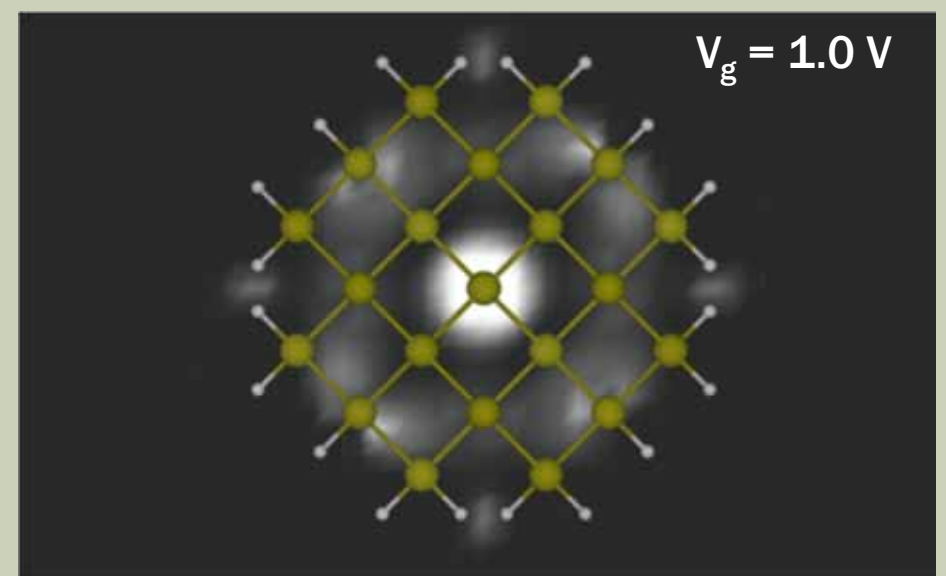
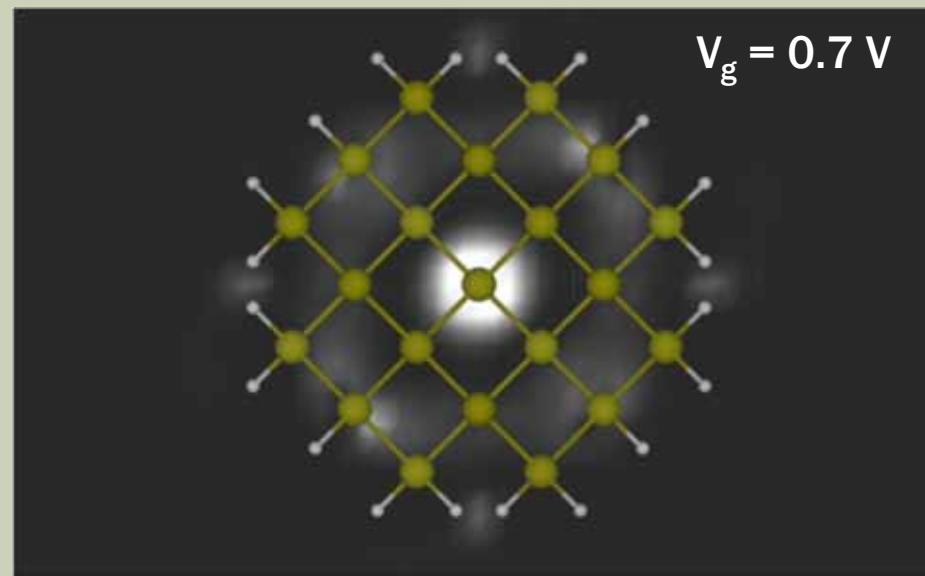
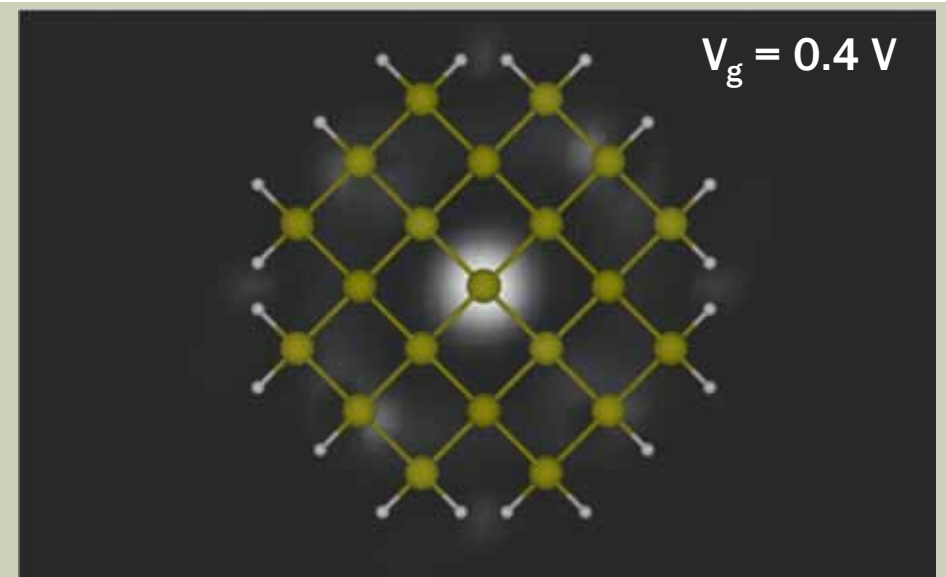
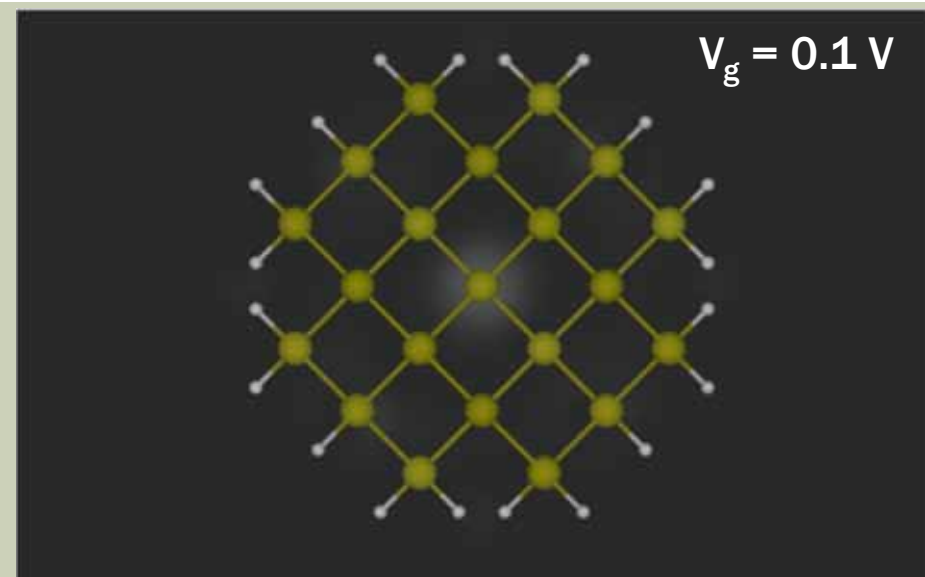


SiNW(111)
 diameter : 4nm
 temperature : 300K



CONTOUR PLOT OF CURRENT DENSITY

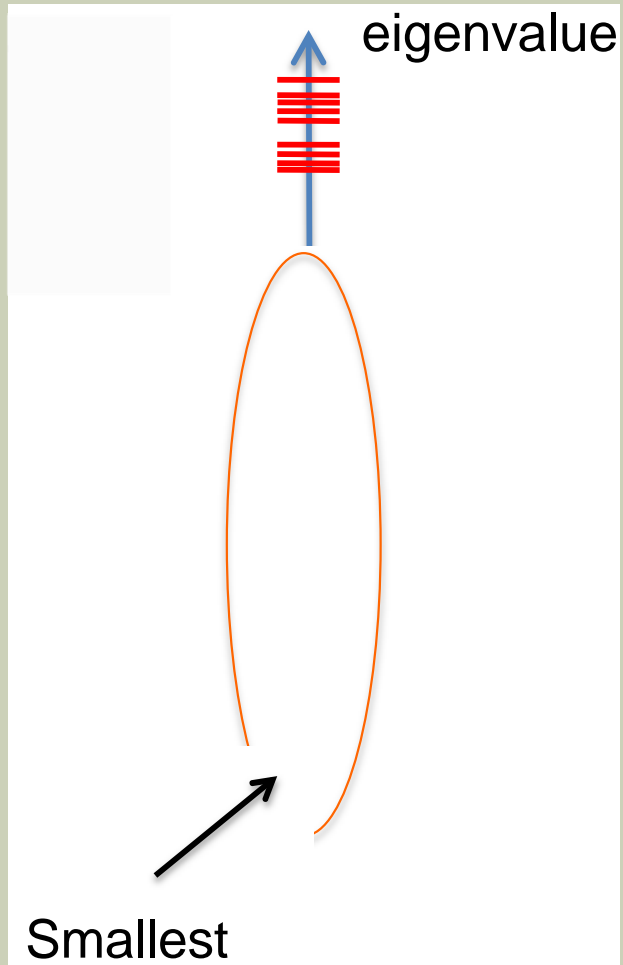
By S. Furuya (VESTA)



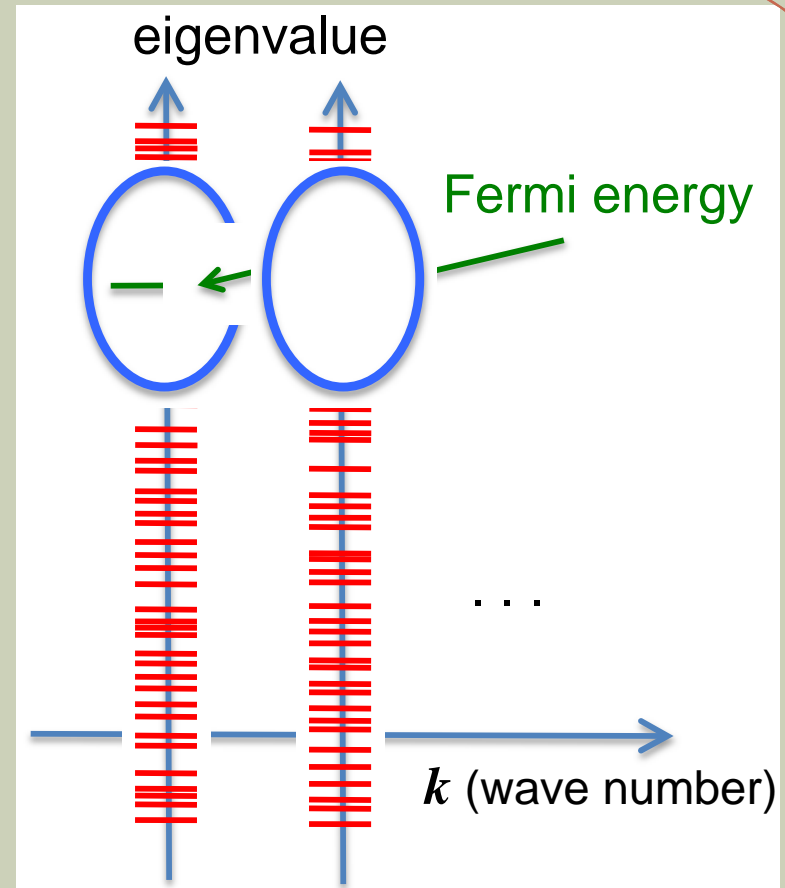
**BAND STRUCTURE
CALCULATIONS WITH
SAKURAI-SUGIURA
METHOD**

二村保徳
筑波大
櫻井研

SCF CALCULATIONS AND BAND CALCULATIONS



SCF calc. → exterior eigenproblem



Band calc. → interior eigenproblem

SAKURAI-SUGIURA METHOD

- A novel eigensolver
 - Suitable for interior eigenproblems
 - Suitable for massively-parallel architectures
- Band structure calculation of 10,000-atom system

CONTOUR INTEGRATION

$$S_k \equiv \frac{1}{2\pi i} \int_{\Gamma} z^k (zI - A)^{-1} V dz$$

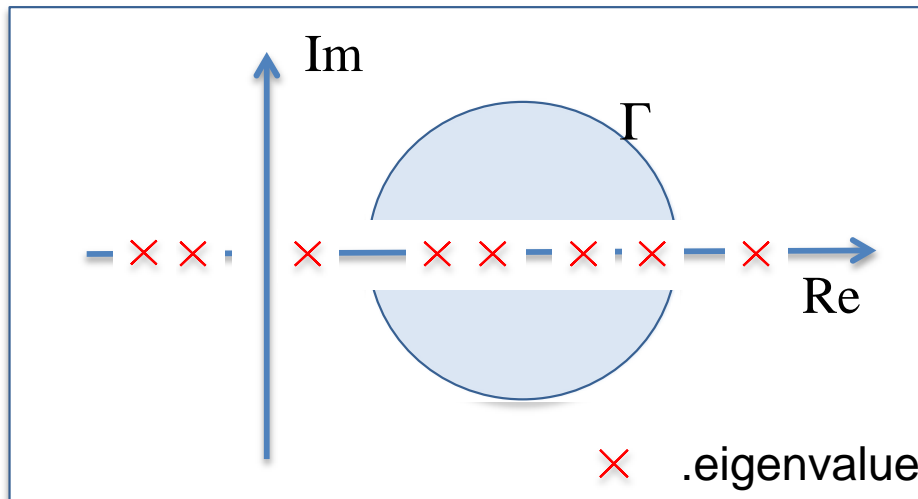
L arbitrary vectors
(linearly independent)

$$S_k, V \in \mathbb{C}^{n \times L}$$

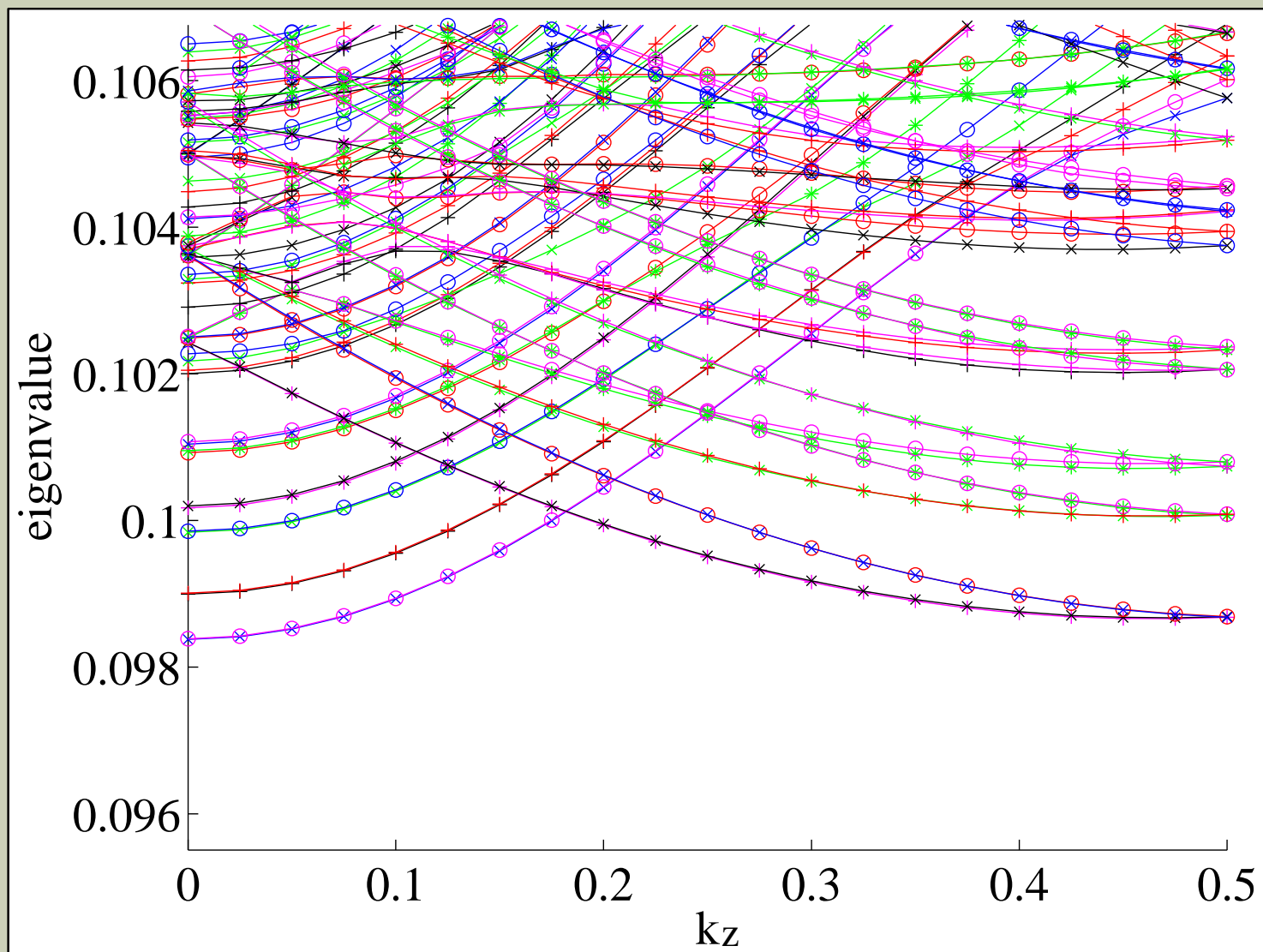
$$k = 0, 1, \dots, M - 1$$

$$L, M \ll n$$

of grid points



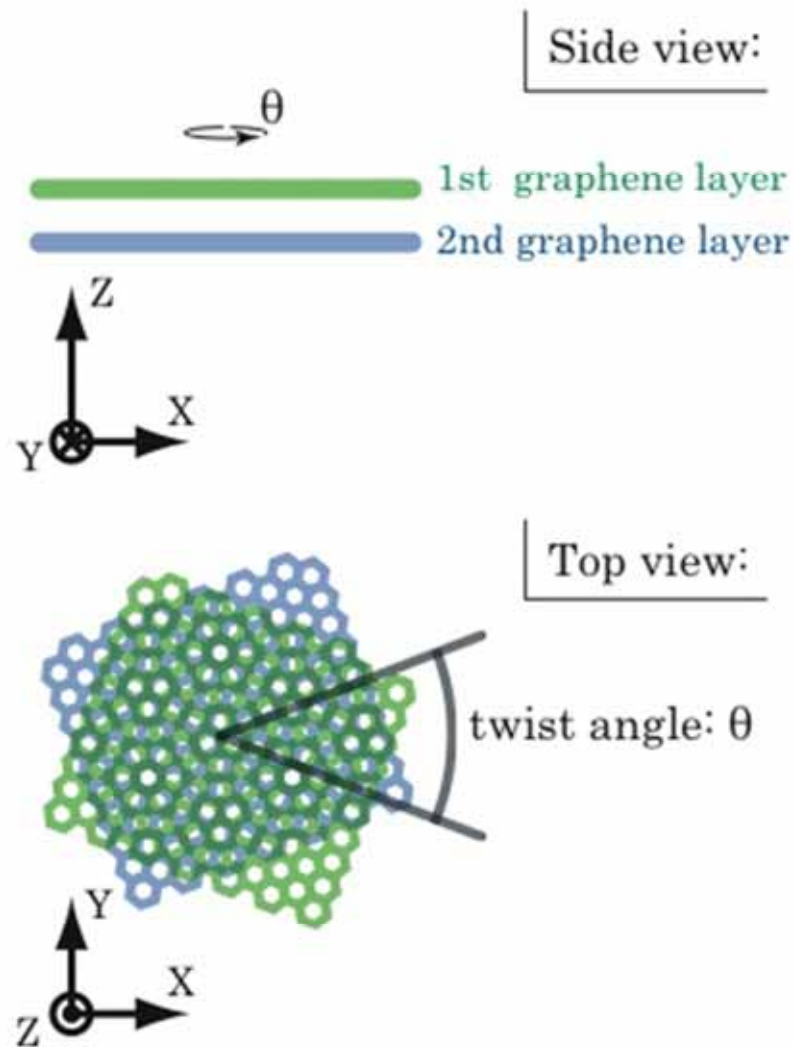
BAND STRUCTURE OF 10,000-ATOM SiNW



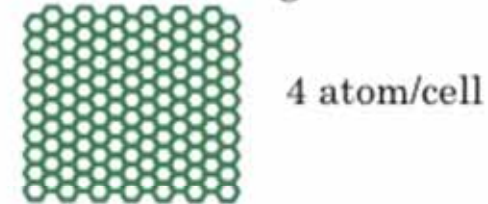
**ATOMIC & ELECTRONIC
STRUCTURES OF
TWISTED BILAYER
GRAPHENE**

内田和之
東大押山研

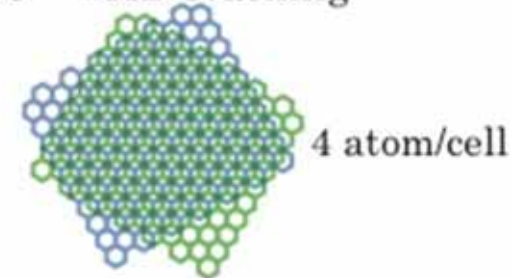
What is Twisted Bilayer Graphene (tBLG)?



■ $\theta=0^\circ$: AA-stacking

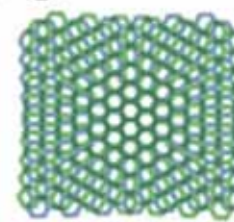


■ $\theta=60^\circ$: AB-stacking

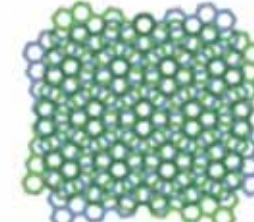


■ $\theta=\text{any value}(\neq 0^\circ, 60^\circ)$

(e.g.) $\theta=6^\circ$



$\theta=20^\circ$



“Moire pattern”

- periodic only for special θ s (\rightarrow this work)
- non-periodic for other θ s

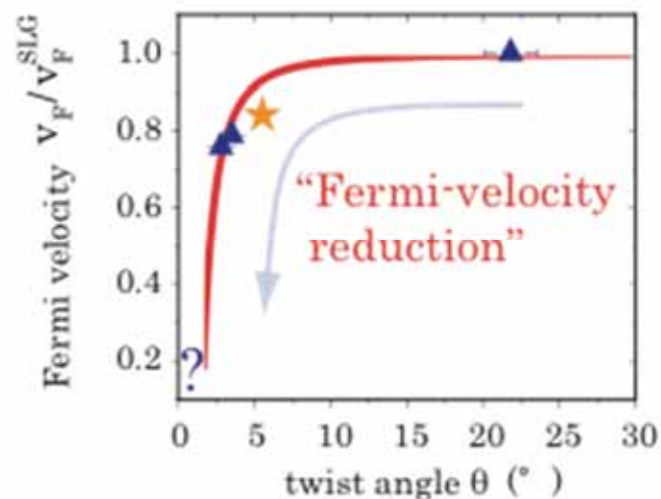
*Assumption: In the above and following, an A-site atom is on the twist axis in each graphene. Other cases will be also discussed later (consideration of lateral translations).

Why tBLG?

① STM+STS for Twisted Graphene Layers

Luican et al. : PRL 106, 126802 (2011)

(STM → Moire → twist angle : θ
STS under magnetic field → Landau levels → Fermi velocity : V_F)



■ Fermi velocity V_F is controlled by changing twist angle θ .

■ consistent with tight-binding calc. for tBLGs

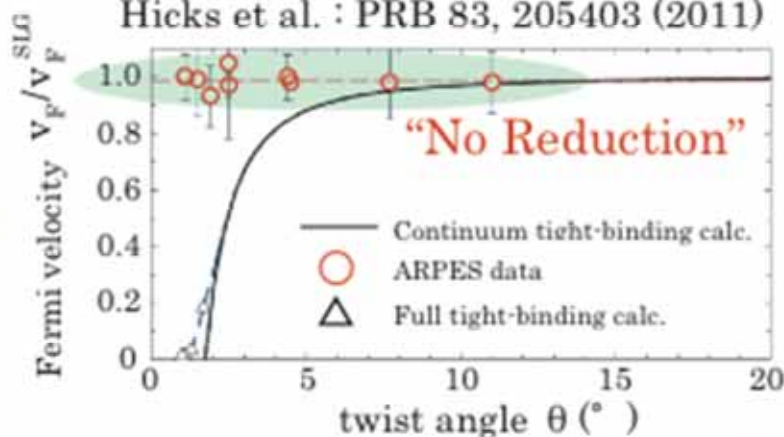
- ▲ : STM+STS
- : continuum tight-binding calc.
- ★ : DFT calc [Trambly et al., Nano Lett 10, 804 (2010)]

However,
controversial



② ARPES

Hicks et al. : PRB 83, 205403 (2011)

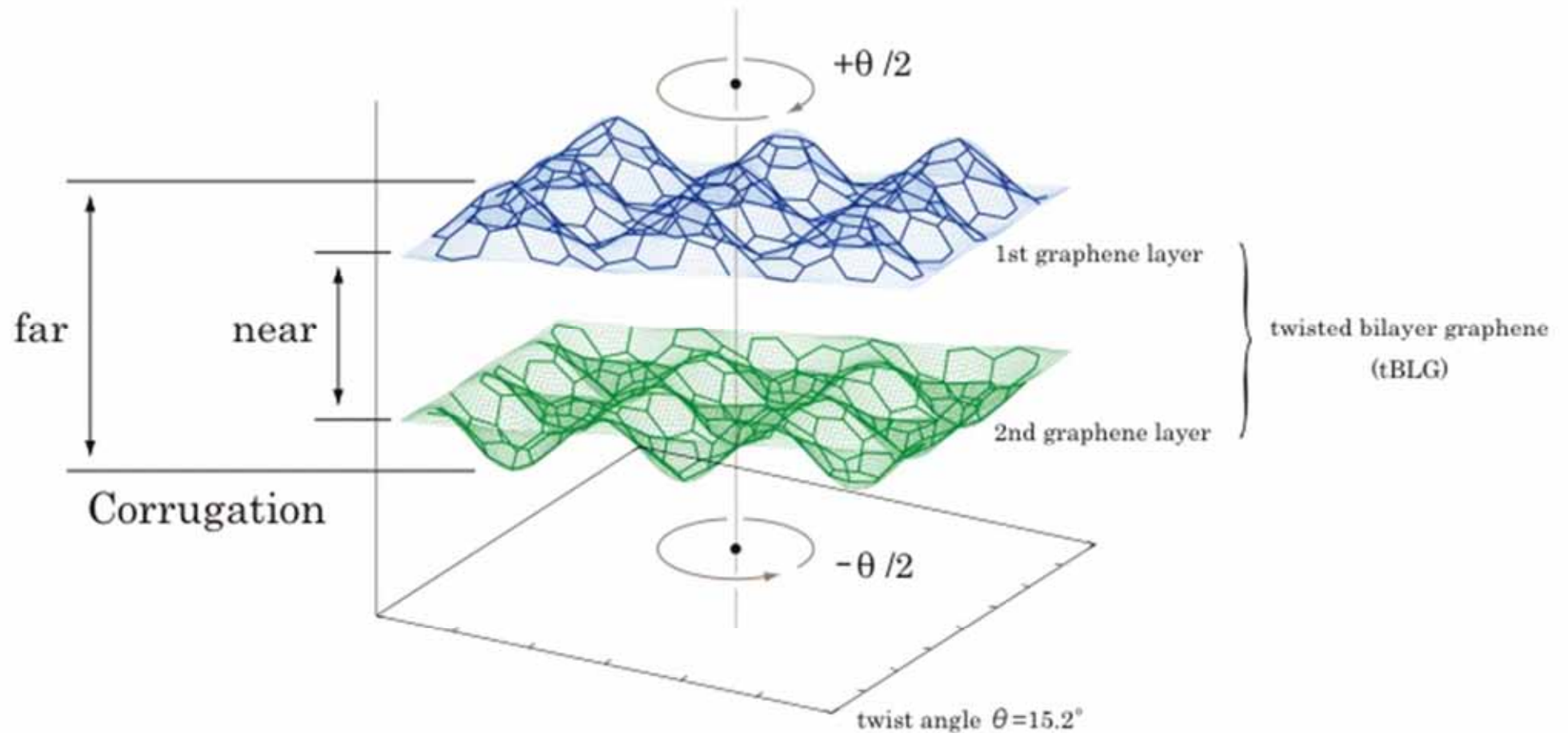


THIS WORK :

We wish to know if the Fermi-velocity reduction is true or not.

→ DFT calculations for tBLGs

Atomic-structure Optimization

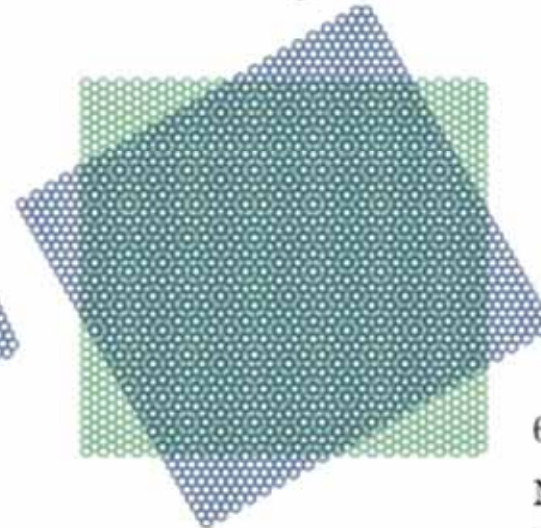
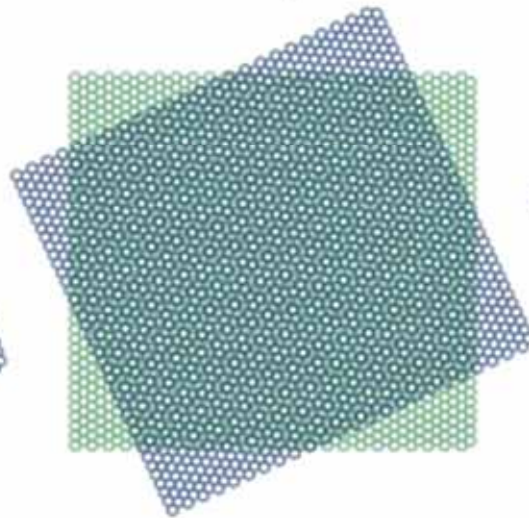
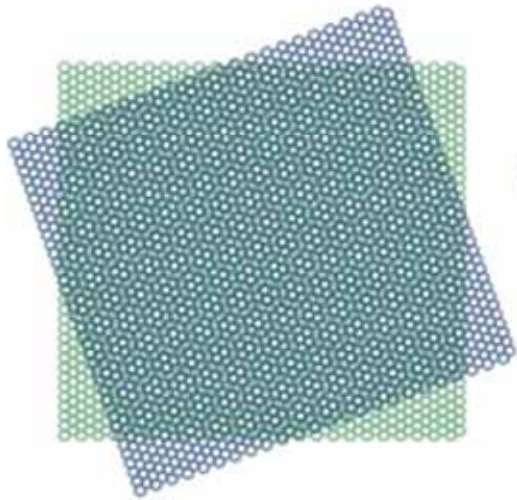


Relation between Corrugations and Stacking Structures

■ twist angle $\theta=30^\circ$

■ twist angle $\theta=25^\circ$

■ twist angle $\theta=20^\circ$



$\theta > \sim 20^\circ$

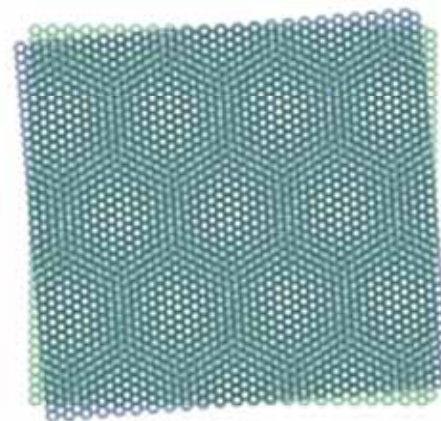
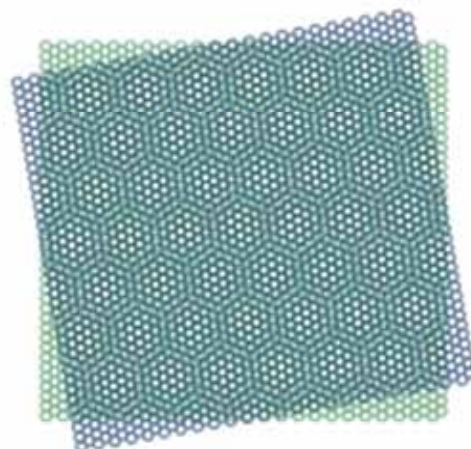
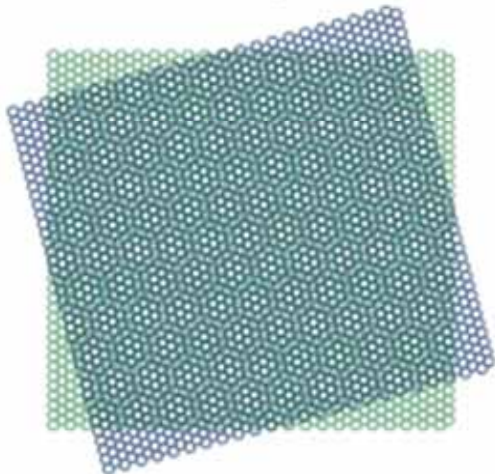
No Corrugation

No locally AA/AB stacked regions

■ twist angle $\theta=15^\circ$

■ twist angle $\theta=10^\circ$

■ twist angle $\theta=5^\circ$



$\theta < \sim 20^\circ$

Corrugation

Locally AA/AB stacked regions exist, and become larger as θ becomes smaller

FIRST-PRINCIPLES
CALCULATIONS OF
STEP STRUCTURES ON
SiC(0001)

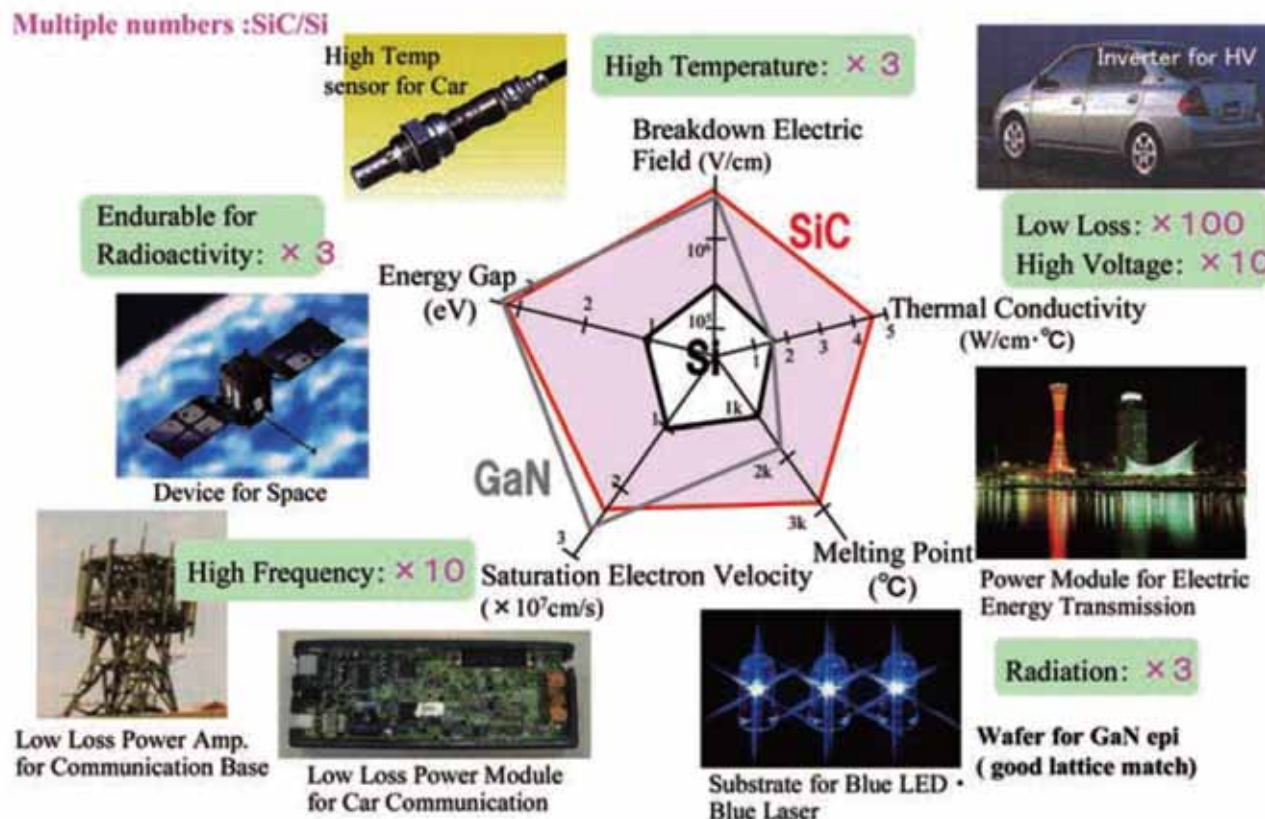
澤田啓介
東大押山研

Silicon-Carbide



- Silicon carbide (SiC) is a hopeful semiconductor for the next generation of **power-electronic-device**.

<http://www.denso.co.jp/ja/aboutdenso/technology/dtr/v16/files/13.pdf>

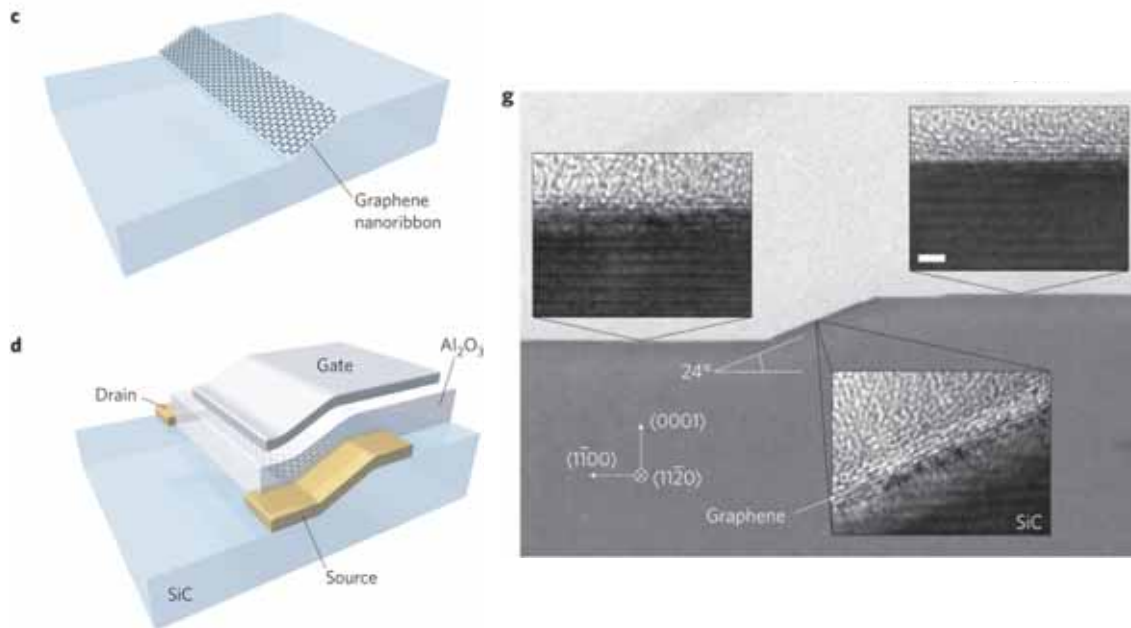


Features and applications of SiC semiconductors.

Template-of-Nanostructures-using-Step-Structures

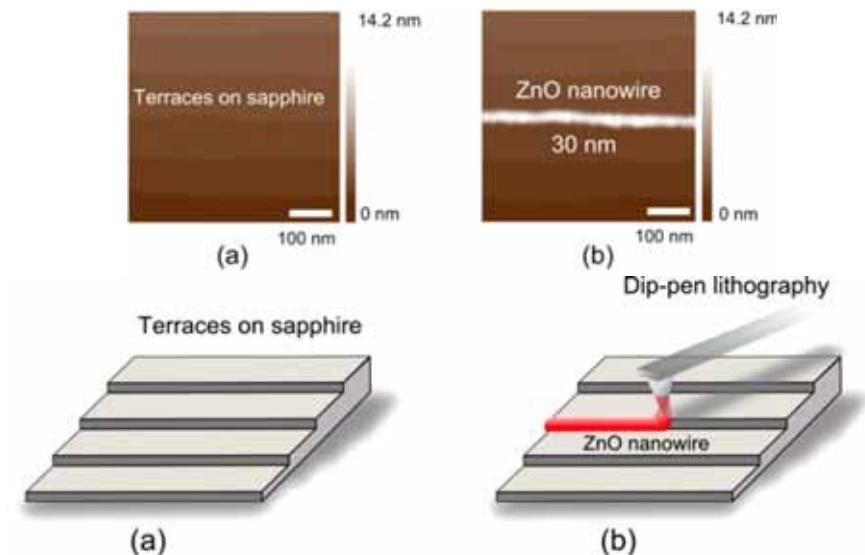
- Step structures can be applicable to the **template-of-nanostructures** such as nano wires, tubes and dots.

M. Sprinkle et al., Nat. Nanotechnol. 5, 727 (2010).



Nanofacet on 4H-SiC(0001) is applied to template of the field-effect transistor using self-organized graphene nanoribbon.

J. Y. Son et al., Electrochem. Solid-State Lett. 14, H397 (2011).



The fabrication of ZnO nanowires is demonstrated on steps of sapphire [Al₂O₃ (0001)] surface.

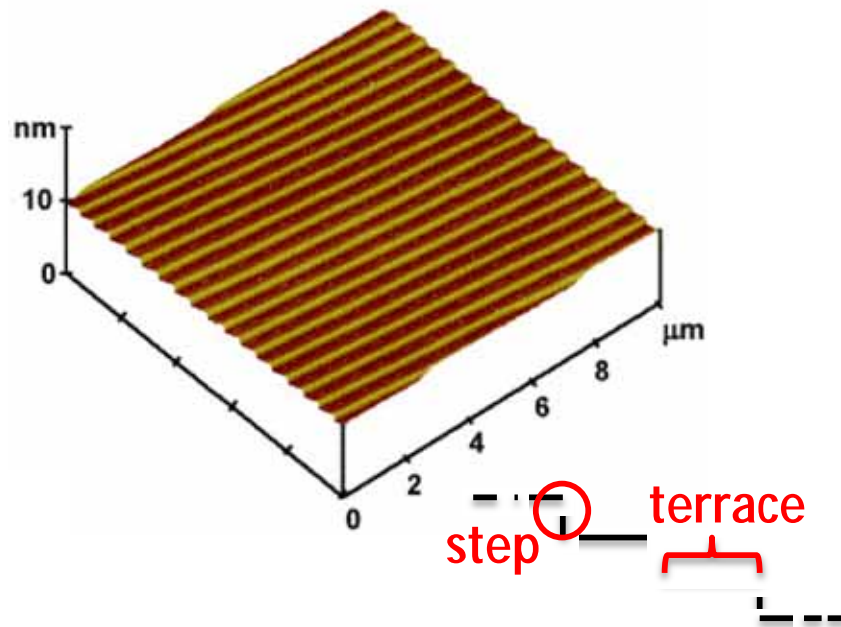
- SiC基板の熱分解によるグラフェン生成

Morphologies-of-Vicinal-Solid-Surface

- It is important to understand structures of surfaces or interfaces on substrates for the device fabrications.
- Vicinal solid surface often shows regularly spaced **step,-terrace-and-nanofacet**-formed by step bunching.

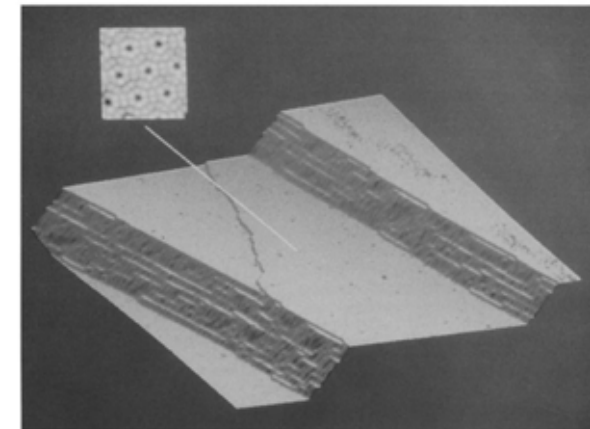
A. Nakajima et al., J. Crystal Growth 278, 437 (2005).

AFM image of 6H-SiC substrate surface.



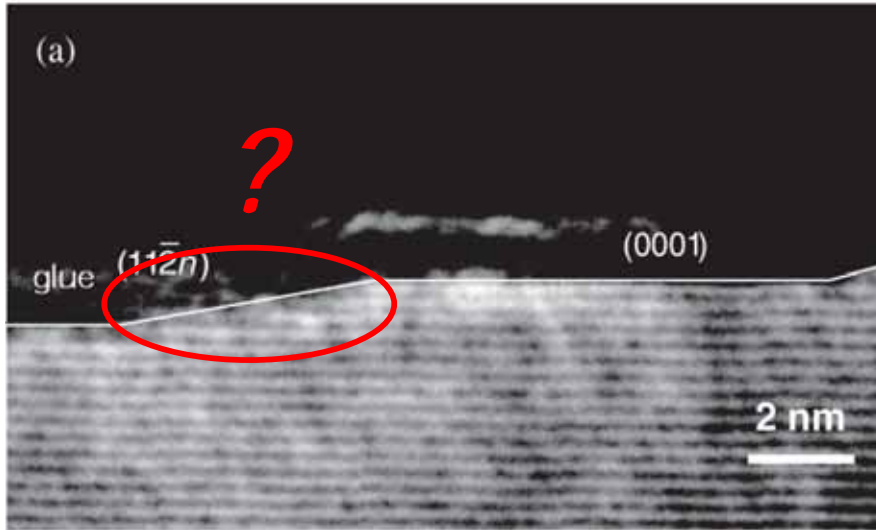
E. D. Williams et al., Surf. Sci. 294, 219 (1993).

STM image of Si(111) surface.

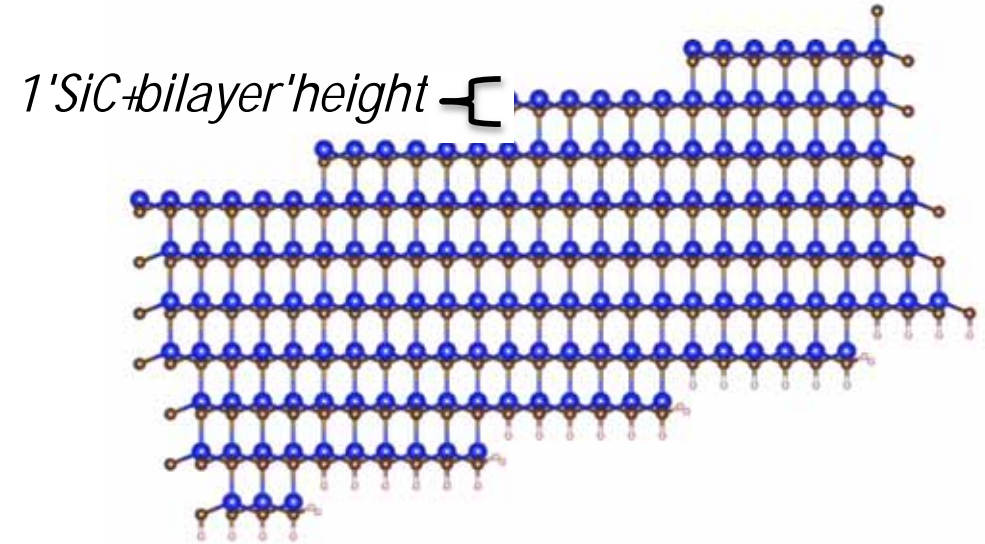


Expected-Simple-Step-Structures

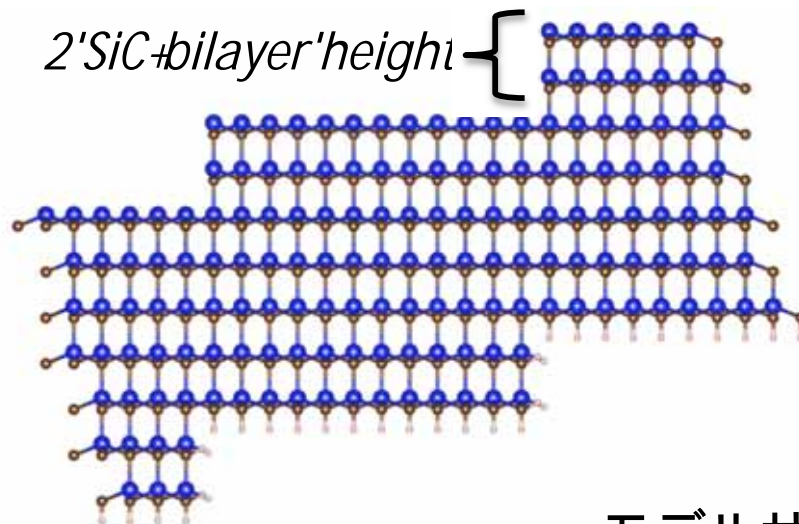
Cross-sectional-TEM-image



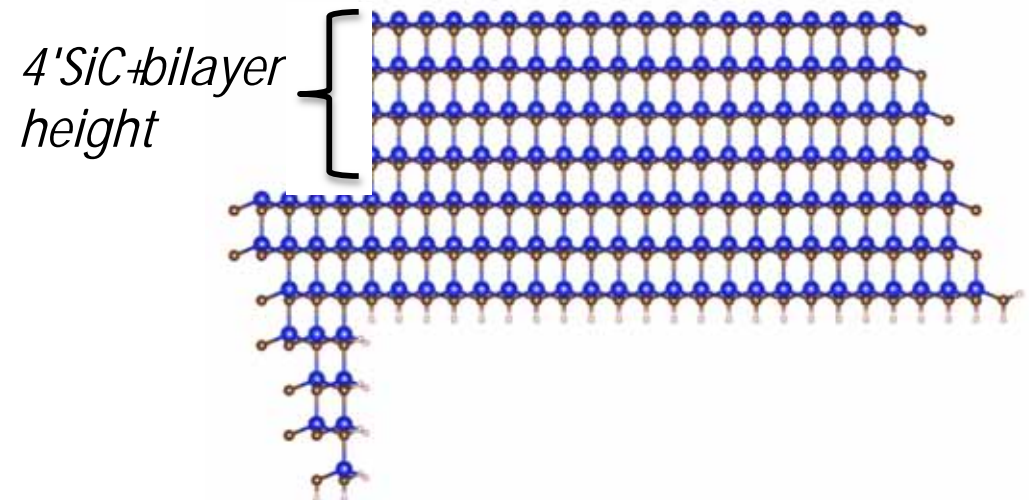
Single-height-step-(SHS)-structure-



Double-height-step-(DHS)-structure-



Quad-height-step-(QHS)-structure



モデルサイズ : 300 ~ 1000原子

まとめと今後

- コンパクトモデルを用いたSiNW-FETの電流電圧特性の計算
 - ゲート電極の効果を第一原理的に扱う（産総研 大谷実）
 - ソース・ドレイン電極も取り入れた第一原理輸送計算
（東大押山研 Zixin Guo、東大渡邊研 笹岡健二）
 - 第一原理デバイスシミュレータ
- 捻れ二層グラフェンの構造と電子状態（東大押山研 内田和之）
- SiC微斜面の構造決定（東大押山研 澤田啓介）
- 機能拡張
 - RSDFT-CPMDの実装（東大押山研 小泉健一、阪大 重田照育）