Large-scale Parallel Numerical Computing Technology Research Team

1. Team members

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2. Research Activities

The Large-scale Parallel Numerical Computing Technology Research Team conducts research and development of a large-scale, highly parallel and high performance numerical software library for K computer. Simulation programs require various numerical techniques to solve linear systems, eigenvalue problems, fast Fourier transforms, and non-linear equations. In order to take advantage of the full potential of K computer, we must select pertinent algorithms and develop a numerical software library based on the concepts of high parallelism, high performance, high precision, resiliency, and scalability.

Our primary mission of this project is to develop a highly parallelized and scalable numerical library on the K computer system, namely KMATHLIB. It comprises several components such as for solving

1) System of linear equations,
2) Eigenvalue problems,
3) Singular value decomposition,
4) Fast Fourier transforms, and
5) Nonlinear equations.

The K-specific topics are also our challenging works as follows;

a) Tofu interconnect,
b) Parallel I/O,
c) Fault detection (soft-error), and
d) Higher accuracy computing.

We are going to complete this project through close collaboration among computational science (simulation), computer science (hardware and software) and numerical mathematics. Our final goal is to establish a fundamental technique to develop numerical software libraries for next generation supercomputer systems based on strong cooperation within AICS.

3. Research Results and Achievements

In this report, we focus on our three projects, 1) development of KMATHLIB, 2) development of EigenExa, and 3) investigation of the FDTD related method. Other activities by each researcher and collaborative works with other AICS teams are summarized in the last two sections, future plans and the publication list.

3.1. KMATHLIB Project

1. **Development of the KMATHLIB for the integration of OSS packages**

Since FY2012-2013, we have developed an integration framework KMATHLIB for number of numerical libraries installed on K computer. KMATHLIB supports a wide range of spectral of a lot numerical libraries, and KMATHLIB-API covers the resource usage from hundred to ten-thousand nodes or up to the whole system.

The schematic of KMATHLIB is depicted in Figure 1. KMATHLIB-API is on the top layer and is accessed by users directly. By taking account of KMATHLIB, we can plug-in favorite OSS like the bottom part of Figure 1. KMATHLIB-API can conceal the differences of API's and data structures, etc.

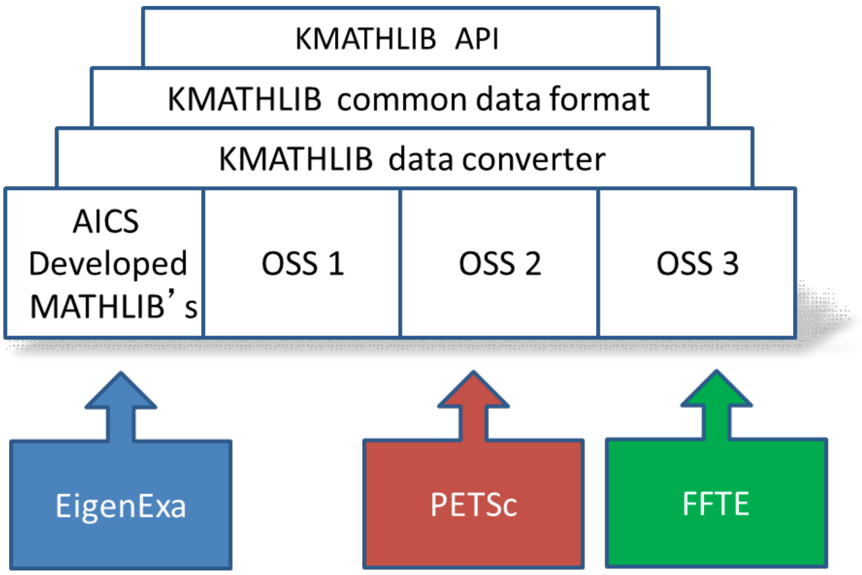

Figure 1. The structure of the KMATHLIB package.

Example of KMATHLIB-API is shown in Figure 2. KMATHLIB-API adopts a modern API style of the standard numerical libraries, like PETSc and FFTW. In order to use the numerical solver plugged in the KMATHLIB package, user needs to proceed four steps.

1. First, the user initiates KMATHLIB.
2. Create a handle of each solver. The handle is bound with several attributes about underlying data and options of the solver function to be called.
3. Then, the user invokes a solver function associated with the handle.
4. Finally, the finalization step must be done before the end of the program.

We have successfully integrated the standard numerical libraries such as LAPACK, ScaLAPACK, EigenExa, PETSc, FFTW, FFTE, and SSL II into the KMATHLIB package.

```
program xxx
use kmath_ls
Vec     x, b
Mat     A
KLS     kls

call Kmath_Initialize( )
call Kmath_LSCreate( MPI_COMM_WORLD, kls, ierr)

call Kmath_LSSolve( kls, A, b, x, ierr)

call Kmath_LSDestroy(kls, ierr)

end program
```

Figure 2. Example code of the KMATHLIB linear solver.

2. Enhancement of the KMATHLIB numerical libraries

Since FY2012-2013, we also have examined the parallel performance of the standard parallel OSS, and we have concluded that most of OSS does not scale on a large number of processors. We decided to develop and enhance the performance of ported OSS. In this report, we present two notable results of KMATH_RANDOM (random number generator) and newly proposed Divide and Conquer method for GEPBs.

a) Enhancement of the KMATH_RANDOM library

In FY2012-2013, we parallelized and integrated an open source real random number generator dSFMT into the KMATHLIB framework, namely KMATH_RANDOM. To support it on K computer, we modified KMATH_RANDOM in FY2013-2014 so that each MPI process restarts to generate real random numbers independently. We have evaluated the KMATH_RANDOM library on K computer. Figure 3 shows the strong scaling behavior of the generator function which successively yields one billion random numbers. Although KMATH_RANDOM performs well, it scales down slightly when the number of nodes is large.

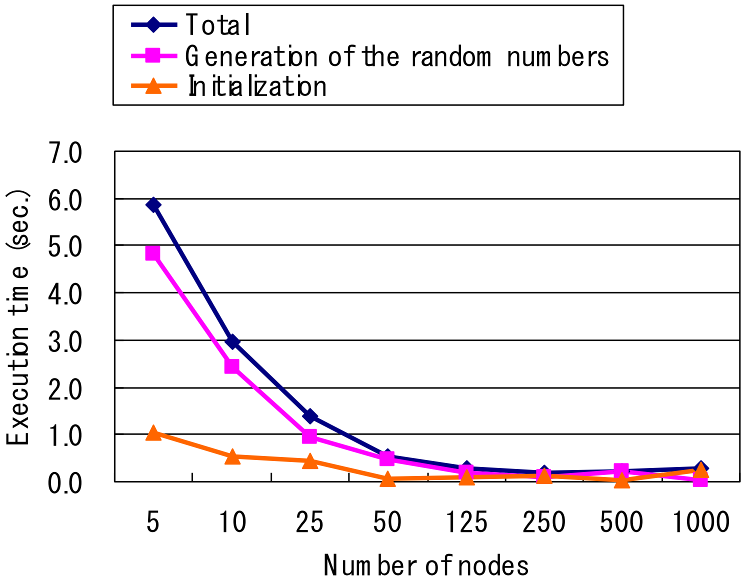

Figure 3. Strong scaling performance of KMATH_RANDOM on K computer.

b) New computational scheme by Divide and Conquer method for GEPBs

As a part of the KMATHLIB project, we have carried out basic research on a development of a solver for generalized eigenvalue problems of banded matrices (GEPBs)

$$A\boldsymbol{w} = \lambda B\boldsymbol{w}.$$

Here, *A* and *B* are N-by-N symmetric band matrices and *B* is positive definite. GEPBs often appear in quantum chemistry and the numerical solver of GEPBs can be used as a part of the solution method for dense symmetric definite GEPs. The faster performance of the solver for GEPB is strongly required. Rough analysis, however, reveals poor parallel performance of the conventional methods (e.g. parallelized versions of DSBGV and DSBGVD in LAPACK). We have proposed an alternative divide and conquer method for GEPBs which is based on the Elsner's idea (1997). The method has advantages that its FLOPS count is smaller than the conventional method and the most of the operations can be parallelized efficiently. Figure 4 shows the residuals $||AW - BW\Lambda||_F / ||A||_F$ obtained by the conventional solver (DSBGV) and a prototype of the proposed method. Here, $W = [\boldsymbol{w}_1, \boldsymbol{w}_2,$

…, $\mathbf{w}_N$], Λ = diag(λ_1, λ_2, ..., λ_N), and (λ_i, $\mathbf{w}_i$) refers to the *i*-th eigenpair. We confirmed that the accuracy of the proposed is comparable to or better than the conventional method.

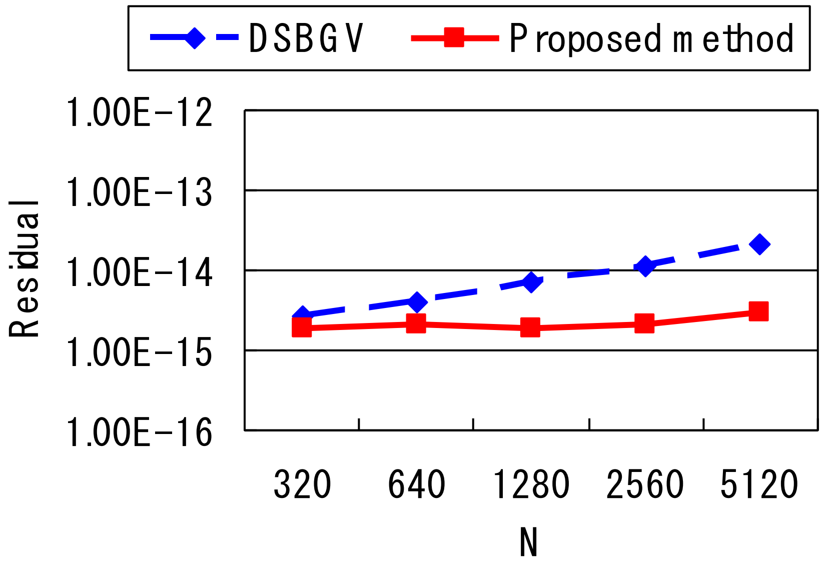

Figure 4. Accuracy test of DSBGV and the implementation of the proposed method.

3.2 EigenExa Project

The EigenExa Project is basically a collaborative work between Prof. Yusaku Yamamoto (formerly Kobe University, presently the University of Electro Communications) granted from 'Development of System Software Technologies for post-Peta Scale High Performance Computing' by JST CREST. We conduct mainly a part of the development of a high performance parallel dense eigenvalue solver, namely EigenExa.

1. Implementation of a Parallel solver

In a last Annual report for FY2012-2013, we reported the progress of this project and the status of the Eigen-K library. This FY2013-2014, the first version of the EigenExa library was released on August 2013 as a successor of Eigen-K. User can download EigenExa from the URL, http://www.aics.riken.jp/labs/lpnctrt/EigenExa_e.html.

From the same URL, we can refer to the poster (Figure 5) which was presented at International Workshop EPASA2014. Figure 5 illustrates the latest performance report and our outstanding achievement of world largest scale eigenvalue computation on K computer.

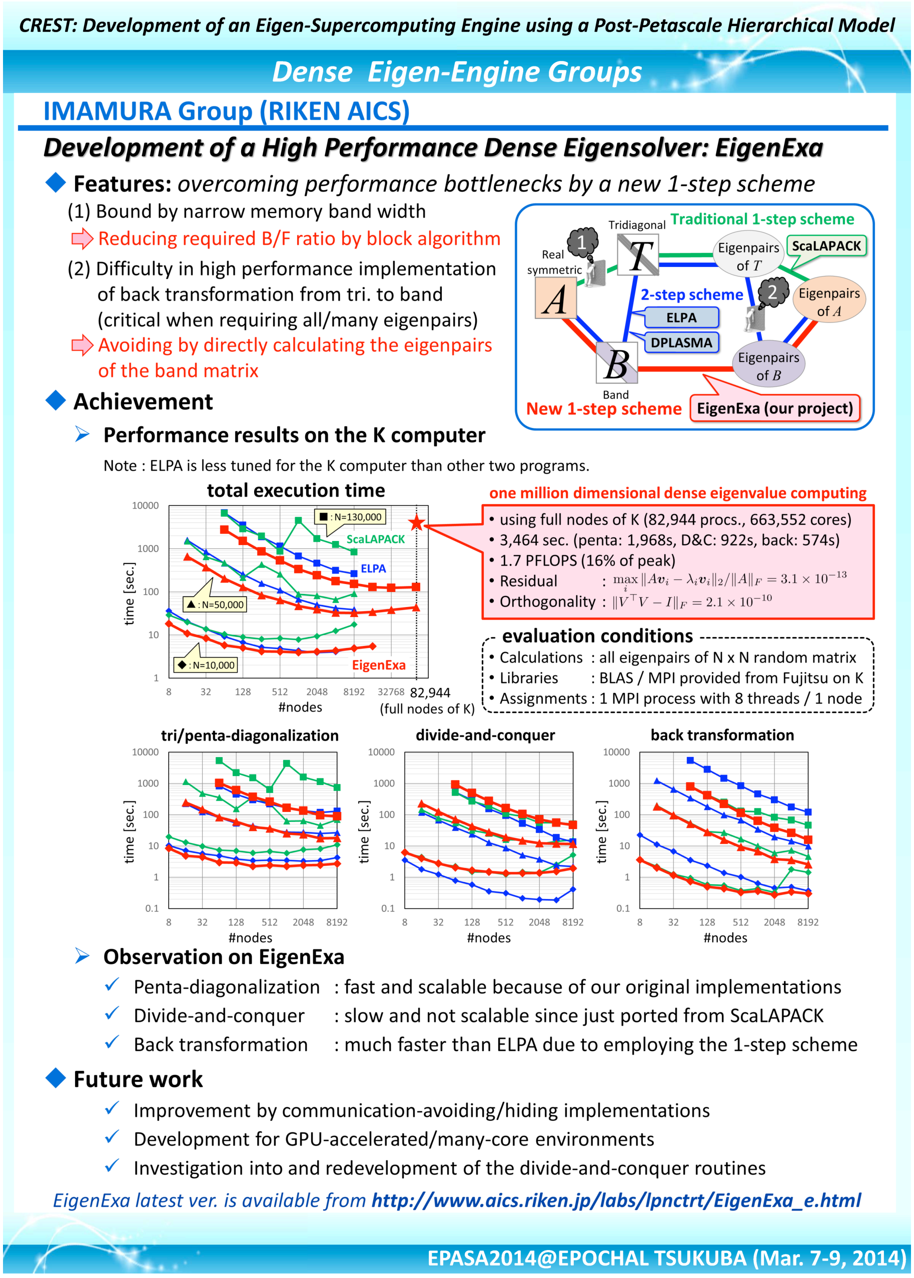

Figure 5. Poster from http://www.aics.riken.jp/labs/lpnctrt/EigenExa_e.html

As you can see the description of "one-million dimensional dense eigenvalue computing", we successfully diagonalized a one-million dimensional random symmetric matrix within one hour by taking account of the full computational nodes of the K computer system. The result shows a favorable properties of the EigenExa library towards the emerging Exa-scale computing era.

2. Theoretical studies and Practices for Communication Avoiding

We have studied basic building blocks of high performance dense eigenvalue solvers. We have analyzed the parallel performance of our solver to construct a performance model for predicting the performance on a post-peta scale system. In this FY2013-2014, we built a prototype model which estimates the parallel execution time of the most dominant part of our scheme, and we examined its accuracy on K computer (see Figure 6).

We have also studied a communication-avoiding algorithm, called TSQR (or CAQR), for computing a tall-skinny QR factorizations, required in both dense and sparse eigenvalue solvers. We evaluated its performance, particularly the strong scaling, by using K computer. In addition, we pointed out its performance bottleneck by constructing a realistic performance model and are now planning to improve the performance.

Finally, we report our two performance evaluations by using the whole FX10 supercomputer system, Oakleaf-FX, supported by "Large-scale HPC Challenge" Project, Information Technology Center, and the University of Tokyo. In the first one, we measured the performance of EigenExa in various conditions; we changed the size of a target matrix, program parameters such as the shape of process grid and combination of the number of MPI processes and that of the OpenMP threads. In the second one, we compared the performance of TSQR/CAQR with other conventional algorithms such as Householder QR. The results of both evaluations are expected to contribute to further performance tuning of each program.

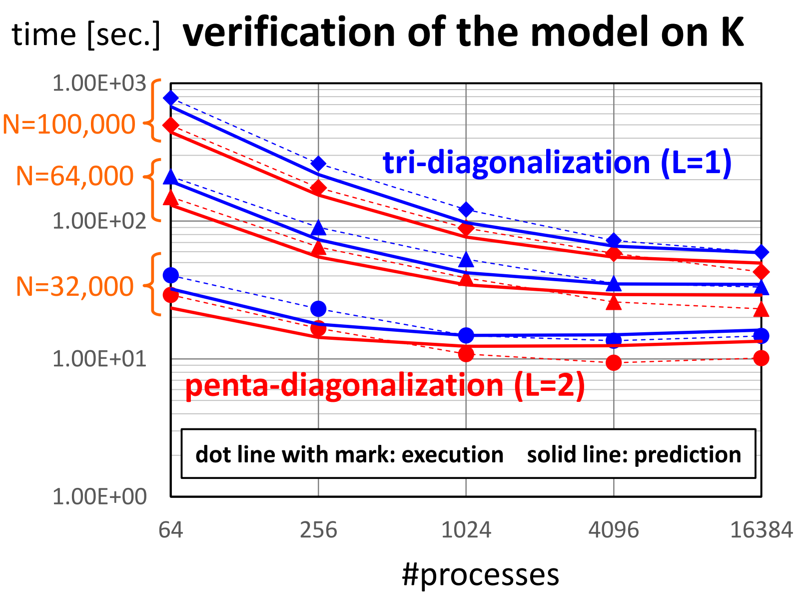

Figure 6. Verification of our performance model on K computer.

3.3 Investigation of the FDTD Related Methods

In order to develop useful mathematical software for K computer, it is significant to investigate general numerical frameworks not focused on particular simulation codes. FDM (Finite Difference Method) and FEM (Finite Element Method) are well-known numerical discretization and time-integration frameworks for scientific simulation codes, which are based on fixed spatial grid coordination. Other numerical principles to be handled on simulation codes are the particle method and the mesh-less methods. In FY2013-2014, we started the study of the FDTD related issues. This subsection illustrates the current status of our studies.

1. **FDTDM (Finite Difference Time Domain Method)**

   Recent public interests center on the electric power problem, such as an electrical generator, a charge device for an electric vehicle, and side effect of an electromagnetic medical device. Finite Difference Time Domain Method (FDTDM) plays an important role in a numerical simulation of such problems. We recognized that it is one of the key issues to investigate a parallel FDTDM software on K computer while most of the standard FDTDM software is commercial. We mainly conduct 1) numerical analysis of FDTDM, and 2) development of open source software based on FDTDM. Since FY2013-2014, we have investigated the feasibility of FDTDM for numerical simulations.

2. **MTDM (Meshless Time Domain Method)**

   Though FDTDM is a strong numerical scheme for time-dependent electromagnetic simulation, it has a strong restriction on the computational domain. There is a lot of progresses to treat problems in a complex shaped domain, for example, finer meshes or adaptive meshes, etc. Generally, meshless method requires no information about geometrical structure, so an enhancement of FDTDM taking consideration of the meshless framework is an alternative for the numerical simulation in a complex-shaped domain. We study the Meshless Time Domain Method (MTDM) based on the Radial Point Interpolation Method (RPIM). Since FY2013-2014, we have studied the numerical analysis for a 2D simulation and the computational accuracy in terms of 'node distribution' and 'base functions', which are thought as important parameters of MTDM. In addition, we have improved the performance of the MTDM code.

4. Schedule and Future Plan

Our three main projects, KMATHLIB, EigenExa, and FDTD are progressing satisfactorily. They are ongoing projects and there will be some update as follows.

1. **KMATHLIB project**

   In order to promote the KMATHLIB package, we need to support a lot of useful plugged-in OSS. We plan to release the first version of the solver of GEBPs for K computer as a part of

KMATHLIB in FY2014-2015. The generalized eigenvalue solver for dense matrices which comprises the solver for banded matrices will be released in FY2015-2016 or later. Another plan is to develop and release a prototype of the Helmholtz equation solver in FY2014-2015 which is collaborated by the SCALE project.

2. EigenExa project

After the first release of the EigenExa library, several application codes adopt to use EigenExa. We got another important mission to maintain EigenExa continuously. Furthermore, we recognize that it is also significant to improve the performance and scalability. As reported in the last section, we struggled with the communication overhead when the number of processors used is quite large. Towards the post K computer system, HPC researchers, especially who are interested in numerical linear algebra, keep watch on the progress of new technologies such as Communication Avoiding, Communication Hiding, and Synchronous Avoiding. We are going to investigate these in the future EigenExa library.

3. FDTD related method

Our must topics for the FDTD related project are to support a 3D simulation by using MTDM and to analyze computational precision in a practical simulation. In fact, an accurate numerical simulation of side-effects from an MRI device to a human-body has a big impact to life science and medical engineering. We need to promote the FDTD related works to an unexplored field in AICS.

4. Other issues

As we described in the annual report FY2012-2013, "Fault tolerance" or "Resilience" is one of the key words for the peta-scale computing. From the viewpoint of the numerical library, we would like to establish a new algorithmic fault detection mechanism and its framework.

5. Publication, Presentation and Deliverables

(1) Journal Papers

[1] Y. Idomura, M. Nakata, S. Yamada, M. Machida, T. Imamura, T. Watanabe, M. Nunami, H. Inoue, S. Tsutsumi, I. Miyoshi, and N. Shida, "Communication overlap techniques for improved strong scaling of gyrokinetic Eulerian code beyond 100k cores on the K-computer", Intl. J. of High Performance Computing Applications (IJHPCA), SAGE publications, Vol. 28, No. 1, pp. 73-86, Feb. 2014. doi: 10.1177/1094342013490973.

(2) Conference Papers

[2] T. Imamura, S. Yamada, and M. Machida, "A High Performance SYMV Kernel on a Fermi-core GPU", High Performance Computing for Computational Science - VECPAR 2012, Lecture Notes in Computer Science, Vol. 7851 (LNCS 7851), pp. 59-71, Springer Verlag, 2013.

[3] S. Yamada, T. Imamura, M. Machida, "Parallel Computing Design for Exact Diagonalization Scheme on Multi-band Hubbard Cluster Models", International Conference on Parallel Computing (ParCo2013), Technische Universiät München, Campus Garching, Munich, Germany, Sep. 10-13, 2013, M. Bader, A. Bode, H.-J. Bungartz, M. Gerndt, G.R. Joubert, F. Peters (eds.): Parallel Computing: Accelerating Computational Science and Engineering (CSE). Advances in Parallel Computing 25, pp. 427-436, IOS Press, 2014.

[4] A. Kuroda, N. Oi, H. Inoue, H. Murai, T. Yamasaki, T. Ohno, T. Imamura, and K. Minami, "Communication Optimization Method on a High-dimensional Mesh/Torus Network for Real Applications --Case Study for the Tofu Network of 'K-computer'--, High Performance Computing Symposium 2014 (HPCS2014), Hitotsubashi-Hall, Tokyo, Jan. 7-8, 2014, IPSJ Symposium Series, Vol. 2014, pp. 97-105, 2013-12-31. (in Japanese)

(3) Invited Talks

[5] T. Imamura, "Roadmap to Eigensolver on a GPU-cluster", GTC Japan 2013, technical session (Research Meeting for GPU Computing organized by GSIC Tokyo Institute of Technology), Tokyo Midtown, Tokyo, Jul. 30, 2013. (in Japanese).

[6] T. Imamura, "Large-scale Eigenvalue solver --from the current status on K computer to upcoming Exa-scale computer ---", Workshop for future HPC (Basic Technologies and Applications), Nagasaki City Library, Nagasaki, Dec. 8-9, 2013. (in Japanese)

[7] T. Imamura, "Communication Avoiding algorithms in numerical linear algebra", 16-th Research Meeting for Matrix Computation Activity Group, JSIAM, the University of Tokyo, Tokyo, Dec. 26, 2013. (in Japanese)

(4) Posters and Presentations

[8] T. Imamura, S. Yamada, and M. Machida, "Eigen-G: GPU-based eigenvalue solver for real-symmetric dense matrices", 10-th International Conference on Parallel Processing and Applied Mathematics (PPAM2014), Polish-Japanese Institute of Information Technology, Warsaw, Poland, Sep. 8-11, 2013. (the paper will be published in Proc., Wyrzykowski, R., Dongarra, J., Karczewski, K., Waśniewski, J. (Eds.), Parallel Processing and Applied Mathematics, Revised Selected Papers, Part I, Lecture Notes in Computer Science, Vol. 8384 (LNCS 8384), Jun. 2014)

[9] T. Imamura, "Automatic Tuning for GPU BLAS kernels", Dagstuhl Seminar, No. 13401 "Automatic Application Tuning for HPC Architectures", Schlöss Dagstuhl, Saarbrucken, Germany, Sep. 29-Oct. 4, 2013. (Oral)

[10] C. Bi, K. Ono, K.-L. Ma, H. Wu, and T. Imamura, "Proper orthogonal decomposition based parallel compression for visualizing big data on the K computer", IEEE Symposium on Large-Scale Data Analysis and Visualization (LDAV 2013), Oct. 13-14, 2013, Atlanta, Georgia, USA, B. Geveci, H. Pfister, and V. Vishwanath (Eds.): Proc. LDAV2013, pp. 121-122, 2013. (Poster)

[11] T. Imamura, S. Yamada, and M Machida, "Strategic study for an O(mega)-core scale parallel eigenvalue solver by using Automatic Tuning technology", 18-th Annual meeting of Japan Society for Computational Engineering and Science, Institute of Industrial Science, the University of Tokyo, Jun. 19-21, 2013, Proc. JSCES, Vol. 18, D-13-5, 2013. (Oral, in Japanese)

[12] N. Sasa, S. Yamada, M. Machida, T. Imamura, and Y. Okuda, "Development and performance evaluation of QPBLAS-GPU", 18-th Annual meeting of Japan Society for Computational Engineering and Science, Institute of Industrial Science, the University of Tokyo, Jun. 19-21, 2013, Proc. JSCES, Vol. 18, D-9-4, 2013. (Oral, in Japanese)

[13] Y. Hirota, and T. Imamura, "Divide and Conquer Method for Generalized Eigenvalue Problems of Banded Matrices", Summer United Workshops on Parallel, Distributed and Cooperative Processing 2013 (SWoPP 2013), Research meeting for Matrix Computation Activity Group, JSIAM, Kitakyushu International Conference Center, Kitakyushu, Jul. 31-Aug. 2, 2013. (Oral, in Japanese)

[14] T. Fukaya, T. Imamura and Y. Yamamoto, "A study on the performance modeling of a dense matrix computation on a massively parallel system", Summer United Workshops on Parallel, Distributed and Cooperative Processing 2013 (SWoPP 2013), Kitakyushu International Conference Center, Kitakyushu, Jul. 31-Aug. 2, 2013, IPSJ SIG Tech. Rep. [High Performance Computing], Vol. 2013-HPC-140, No. 41, pp. 1-8, 2013-07-24. (Oral, in Japanese)

[15] Y. Ohi, T. Tatsuno, and S. Ikuno, "Numerical investigation for stability of electromagnetic field analysis using shape functions based on RPIM", JSIAM 2013 annual meeting, ACROS Fukuoka, Fukuoka, Sep. 9-11, 2013. (Oral, in Japanese)

[16] T. Fukaya, T. Imamura and Y, Yamamoto, "Performance evaluation and estimation of a dense symmetric eigensolver on the K computer", JSIAM annual meeting 2013, ACROS Fukuoka, Fukuoka, Sep. 2013. (Oral, in Japanese)

[17] T. Fukaya, Y. Yamamoto, and T. Imamura, "Performance evaluation of a tall and skinny QR factorization on a large-scale parallel system", 11-th Computational Mathematics Conference, Blanc Art Misasa, Tottori, Nov. 3-4, 2013. (Oral, in Japanese)

[18] Y. Hirota, "On the Divide and Conquer Method for Generalized Eigenvalue Problems and an Extention to the Problems of Banded Matrices", 11th Computational Mathematics Conference, Blanc Art Misasa, Tottori, Nov. 3-4, 2013. (Oral, in Japanese)

[19] Y. Ohi, Y. Fujita, T. Itoh, H. Nakamura, and S. Ikuno, "Speed Up of Shape Function Generation in Meshless Time Domain Method", 23-rd International Toki Conference (ITC-23)

on Large-scale Simulation and Fusion Science, National Institute for Fusion Science, Toki, Japan, Nov. 18-21, 2013. (Poster)

[20] T. Imamura, "Research Activities in AICS towards post Peta-scale Numerical Libraries", 4-th AICS International Symposium, Kobe, Japan, Dec. 2-3, 2013. (Oral)

[21] Y. Ohi, T. Itoh, and S. Ikuno, "Speedup for Generation of Shape Function in Meshless Time Domain Method – Application for a circular sector domain having constant curvature", Nonlinearity/ Visualization-Section Meeting in Workshop on developments of simulation methods for plasma-wall interaction 2013, National Institute for Fusion Science, Toki, Dec. 9-10, 2013. (Oral, in Japanese)

[22] T. Fukaya, Y. Yamamoto and T. Imamura, "A TSQR algorithm based on the Gram-Schmidt orthogonalization and its performance evaluation", 16-th Research Meeting for Matrix Computation Activity Group, JSIAM, the University of Tokyo, Tokyo, Dec. 26, 2013. (Oral, in Japanese)

[23] K. Okada, Y. Okamoto, and T. Imamura, "Optimization of the format of input matrix for CRS-based Matrix-Vector multiplication on a multi-GPU environment", High Performance Computing Symposium 2014 (HPCS2014), Hitotsubashi-Hall, Tokyo, Jan. 7-8, 2014, IPSJ Symposium Series, Vol. 2014, pp. 28, 2013-12-31. (Poster, in Japanese)

[24] K. Shirosawa, T. Imamura, and Y. Okamoto, "Thread parallelization of a successive band reduction method for real-symmetric matrices on a multi-core CPU", High Performance Computing Symposium 2014 (HPCS2014), Hitotsubashi-Hall, Tokyo, Jan. 7-8, 2014, IPSJ Symposium Series, Vol. 2014, pp. 29, 2013-12-31. (Poster, in Japanese)

[25] X. Lin, T. Imamura, and Y. Okamoto, "Performance tuning strategy for GEMV kernels by using d-Spline function", High Performance Computing Symposium 2014 (HPCS2014), Hitotsubashi-Hall, Tokyo, Jan. 7-8, 2014, IPSJ Symposium Series, Vol. 2014, pp. 30, 2013-12-31 (Poster, in Japanese)

[26] T. Imamura, and Y. Hirota, "Communication Avoiding-hiding and Auto-tuning for Extreme-scale Eigensolver", SIAM Conference on Parallel Processing for Scientific Computing (PP14), Portland, OR, USA, Feb. 20, 2014. (Oral)

[27] T. Fukaya, and Y. Yamamoto, "Auto-tuning Tall and Skinny QR Factorization", SIAM Conference on Parallel Processing for Scientific Computing (PP14), Portland, OR, USA, Feb. 20, 2014. (Oral)

[28] R. Tamura, T. Imamura, and Y. Nakatani, "Study for TSQR algorithm by Fully Offload model onto a GPU", 143-th SIGHPC IPSJ (HPC143), Ae-no-Kaze, Wakura-Onsen, Mar. 3-4, 2014, IPSJ SIG Tech. Rep. [High Performance Computing], Vol. 2014-HPC-143, No. 21, pp. 1-7, 2014-02-24. (Oral)

[29] T. Sakurai, S.-L. Zhang, T. Imamura, Y. Yamamoto, Y. Kuramashi, and T. Hoshi, CREST project, "Development of an Eigen-Supercomputing Engine using a Post-Petascale Hierarchical

Model", International Workshop on Eigenvalue Problems: Algorithms; Software and Applications, in Petascale Computing (EPASA 2014), Epochal Tsukuba, Tsukuba, Japan, Mar. 7-9, 2014. (Oral)

[30] T. Sakurai, S.-L. Zhang, T. Imamura, Y. Yamamoto, Y. Kuramashi, and T. Hoshi, CREST project, "Development of an Eigen-Supercomputing Engine using a Post-Petascale Hierarchical Model", EPASA2014, Tsukuba, Japan, Mar. 7-9, 2014. (Poster)

[31] T. Imamura, and Y. Yamamoto, "CREST: Dense Eigen-Engine Groups", EPASA2014, Tsukuba, Japan, Mar. 7-9, 2014. (Poster)

[32] Y. Hirota, and T. Imamura, "Divide and Conquer Method for Computing Generalized Eigenvalues of Banded Matrices", EPASA2014, Tsukuba, Japan, Mar. 7-9, 2014. (Poster)

[33] T. Fukaya, Y. Yamamoto, and T. Imamura, "An overview of parallel algorithms for tall-skinny QR factorizations", EPASA2014, Tsukuba, Japan, Mar. 7-9, 2014. (Poster)

[34] Y. Yanagisawa, Y. Nakatsukasa, and T. Fukaya, "Cholesky-QR and Householder-QR factorizations in nonstandard inner product spaces", EPASA2014, Tsukuba, Japan, Mar. 7-9, 2014. (Poster)

[35] T. Imamura, "Development of a high performance eigenvalue solver on third generation NVIDIA GPU's", Material Design through Computics: Complex Correlation and Non-equilibrium Dynamics, Scientific Research on Innovative Areas, a MEXT Grant-in-Aid Project FY2010-2014, 2-nd Research Meeting in FY2013-2014, the University of Tokyo, Tokyo, Mar. 11, 2014. (Oral, in Japanese)

[36] T. Imamura, "Automatic-tuning for CUDA-BLAS kernels by Multi-stage d-Spline Pruning Strategy", 2014 Conference on Advanced Topics and Auto Tuning in High Performance Scientific Computing, (2014 ATAT in HPSC), National Taiwan University, Taipei, Taiwan, Mar. 14-15, 2014. (Oral)

[37] T. Fukaya, "A Communication-Avoiding Algorithm for the Gram-Schmidt Orthogonalization", 2014 Conference on Advanced Topics and Auto Tuning in High Performance Scientific Computing, (2014 ATAT in HPSC), National Taiwan University, Taipei, Taiwan, Mar. 14-15, 2014. (Oral)

[38] Y. Hirota, and T. Imamura, "On Numerical Solution Methods for the Secular Equation Appearing in Divide and Conquer Method for Generalized Eigenvalue Problems", 10th Joint Symposium of JSIAM Activity Groups, Kyoto University, Kyoto, Mar. 19-20, 2014. (Oral, in Japanese)

(5) Patents and Deliverables

[39] EigenExa, http://www.aics.riken.jp/labs/lpnctrt/EigenExa_e.html