



Construction of Predictive Neuron Models

using Large Scale Data Assimilation

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Overview









Building neuron models with data assimilation:

- Multichannel conductance model
- Assimilation of electrophysiological data
- Model validation through prediction
- Bayesian inference boost single valued model

DA for conditioning neural networks:

- Adaptive Bioelectronics
 ⇒ New therapies for cardiorespiratory disease
- Control of switching between chaotic attractors to produce specific motor patterns e.g. gaits
 ⇒ Validation of the command neuron hypothesis

Neuron: a current driven non-linear oscillator





"for their discoveries concerning the ionic mechanisms involved in excitation and inhibition in the peripheral and central portions of the nerve cell membrane"





Sir John Carew Eccles

Alan Lloyd Hodgkin Andrew Fielding Huxley





Hodgkin-Huxley model:

$$C\dot{V} = \overline{g}_{Na}m^{3}h(E_{K} - V) + \overline{g}_{K}n^{4}(E_{Na} - V) + g_{L}(E_{L} - V) + I$$

$$\tau_{m}\dot{m} = m_{\infty} - m(t)$$

$$\tau_{h}\dot{h} = h_{\infty} - h(t)$$

$$\tau_{n}\dot{n} = n_{\infty} - n(t)$$

Kinetics of ionic gates





Activation

Dynamics of the Potassium-activation gate:





5 model parameters

Sodium activation and inactivation gates:

10 model parameters

Ion channels are highly specialized proteins



Function	Protein name (<u>associated gene</u>)								
Delayed rectifier Slowly activating or non-inactivating	K_v α1.x - Shaker-related: K_v 1.1 (<u>KCNA1</u>), K_v 1.2 (<u>KCNA2</u>), K_v 1.3 (<u>KCNA3</u>), K_v 1.5 (<u>KCNA5</u>), K_v 1.6 (<u>KCNA6</u>), K_v 1.7 (<u>KCNA7</u>), K_v 1.8 (<u>KCNA10</u>) K_v α2.x - Shab-related: K_v 2.1 (<u>KCNB1</u>), K_v 2.2 (<u>KCNB2</u>) K_v α3.x - Shaw-related: K_v 3.1 (<u>KCNC1</u>), K_v 3.2 (<u>KCNC2</u>) K_v α7.x: K_v 7.1 (<u>KCNQ1</u>) - <u>KvLQT1</u> , K_v 7.2 (<u>KCNQ2</u>), K_v 7.3 (<u>KCNQ3</u>), K_v 7.4 (<u>KCNQ4</u>), K_v 7.5 (<u>KCNQ5</u>) K_v α10.x: K_v 10.1 (<u>KCNH1</u>)								
A-type potassium channel rapidly inactivating	K _v α1.x - Shaker-related: K _v 1.4 (<u>KCNA4</u>) K _v α3.x - Shaw-related: K _v 3.3 (<u>KCNC3</u>), K _v 3.4 (<u>KCNC4</u>) K _v α4.x - Shal-related: K _v 4.1 (<u>KCND1</u>), K _v 4.2 (<u>KCND2</u>), K _v 4.3 (<u>KCND3</u>)								
Outward rectifying passes current more easily outwards	K,α10.x: K _v 10.2 (<u>KCNH5</u>)								
Inward rectifying passes current more easily inwards	K _v α11.x: K _v 11.1 (<u>KCNH2</u>), K _v 11.2 (<u>KCNH6</u>), K _v 12.3 (<u>KCNH7</u>)								
Slowly activating	K _v α12.x: K _v 12.1 (<u>KCNH8</u>), K _v 12.2 (<u>KCNH3</u>), K _v 12.3 (<u>KCNH4</u>)								
Modifier / silencer heterotetramerize with K _v α2 family members to form conductive channels									

Individual ion channels have specific threshold voltages and kinetics.

The 9 ion channel conductance model



$$C_m \frac{dV}{dt} = -J_{ion} + \frac{I_{app} + V/R_s}{area}$$

where:

$$-J_{ion} = J_{NaT} + J_{NaP} + J_{K1} + J_{K2} + J_{K3} + J_{CaL} + J_{CaT} + J_{HCN} + J_{HCN$$

and $C_m = C / area$ membrane capacitance per unit area.

 $J_{ion} = I_{ion} / area$ ion current density

ID	Channel	Current density	Nominal conductance
		3. (
NaT	Fast and transient Na^+ current	$J_{NaT} = g_{NaT}m^{3}h(E_{Na} - V)$	$g_{NaT} = 110 \text{mS.cm}^{-2}$
NaP	Persistent Na ⁺ current	$J_{NaP} = g_{NaP}m(E_{Na} - V)$	$g_{NaP} = 0.064 \text{mS.cm}^{-2}$
K1	Transient depolarization activated K ⁺ current	$J_{K1} = g_{K1}m^4(E_K - V)$	$g_{K1} = 5 \text{mS.cm}^{-2}$
K2	Rapidly inactivating K ⁺ current (A current)	$J_{K2} = g_{K2}m^4h(E_K - V)$	$g_{K2} = 12 \text{mS.cm}^{-2}$
K3	Ca ²⁺ activated K ⁺ current	$J_{K3} = g_{K3}m(E_K - V)$	$g_{K3}=9.1$ mS.cm ⁻²
CaL	High threshold Ca ²⁺ current	$J_{CaL} = \rho m^2 J_{Ca}$	-
CaT	Low threshold Ca ²⁺ current	$J_{CaT} = m^2 h J_{Ca}$	-
HCN	Hyperpolarization-activated cation current	$J_{HCN} = g_{HCN}h(E_{HCN} - V)$	$g_{HCN} = 0.092 \text{mS.cm}^{-2}$
Leak	Leakage channels (K & Na)	$J_L = g_L(E_L - V)$	$g_L=0.066 {\rm mS.cm^{-2}}$

Calcium current prefactor:

$$J_{Ca} = \left(\frac{g_{out} - g_{in} \exp(V/V_T)}{\exp(V/V_T) - 1}\right) \times V$$

Goldman-Hodgkin-Katz equation

 J_{L}

12 state variables, 71 parameters 12 coupled non linear differential equations



$$\begin{aligned} & \text{Voltage}: dy_1/dt = ((p_2y_2^3y_3 + p_3y_4)(p_4 - y_1) + (p_5y_5^4 + p_6y_6^4y_7 + p_7y_8)(p_8 - y_1) \\ &\quad + (p_{71}y_9^2 + p_{72}y_{10}^2y_{11})19.2970673(p_{11} - 0.0001\exp(y_1/13))/\text{GHK}(y_1) \\ &\quad + p_9(p_{10} - y_1) + p_{12}y_{12}(-43 - y_1) + I_{inj}/p_{13})/p_1 + u(t)(V_{data}(t) - y_1) \end{aligned} \\ & \text{NaT}, m: dy_2/dt = 0.5(1 + \tanh((y_1 - p_{14})/p_{15}) - 2y_2)/(p_{17} + p_{18}(1 - \tanh^2((y_1 - p_{14})/p_{16}))) \\ & \text{NaT}, h: dy_3/dt = 0.5(1 + \tanh((y_1 - p_{19})/p_{20}) - 2y_3)/(p_{22} + p_{23}(1 - \tanh^2((y_1 - p_{19})/p_{21}))) \end{aligned} \\ & \text{NaP}, m: dy_4/dt = 0.5(1 + \tanh((y_1 - p_{24})/p_{25}) - 2y_4)/(p_{27} + p_{28}(1 - \tanh^2((y_1 - p_{24})/p_{26}))) \\ & \text{K1}, m: dy_5/dt = 0.5(1 + \tanh((y_1 - p_{29})/p_{30}) - 2y_5)/(p_{32} + p_{33}(1 - \tanh^2((y_1 - p_{29})/p_{31}))) \\ & \text{K2}, m: dy_6/dt = 0.5(1 + \tanh((y_1 - p_{34})/p_{35}) - 2y_6)/(p_{37} + p_{38}(1 - \tanh^2((y_1 - p_{34})/p_{36}))) \\ & \text{K2}, h: dy_7/dt = 0.5(1 + \tanh((y_1 - p_{39})/p_{40}) - 2y_7)/(p_{42} + p_{44} + 0.5(1 - \tanh(y_1 - p_{39}))) \\ & \cdot (p_{43}(1 - \tanh^2((y_1 - p_{39})/p_{41})) - p_{44})) \end{aligned} \\ & \text{K3}, m: dy_8/dt = 0.5(1 + \tanh((y_1 - p_{50})/p_{51}) - 2y_9)/(p_{53} + p_{54}(1 - \tanh^2((y_1 - p_{50})/p_{52}))) \end{aligned} \\ & \text{CaL}, m: dy_{10}/dt = 0.5(1 + \tanh((y_1 - p_{55})/p_{56}) - 2y_{10})/(p_{58} + p_{59}(1 - \tanh^2((y_1 - p_{55})/p_{57}))) \end{aligned}$$

 $\begin{aligned} \mathbf{CaL}, h: \ dy_{11}/dt = & 0.5(1 + \tanh((y_1 - p_{60})/p_{61}) - 2y_{11})/(p_{64} + p_{65}(1 + \tanh((y_1 - p_{60})/p_{62}))) \\ & \cdot (1 - \tanh((y_1 - p_{60})/p_{63}))(1 - \tanh(y_1 - p_{60})\tanh((1/p_{62} + 1/p_{63})(y_1 - p_{60})))) \\ & /(1 + \tanh((y_1 - p_{60})/p_{62})\tanh((y_1 - p_{60})/p_{63}))) \end{aligned}$

 $\mathbf{HCN}, h: \ dy_{12}/dt = 0.5(1 + \tanh((y_1 - p_{66})/p_{67}) - 2y_{12})/(p_{69} + p_{70}(1 - \tanh^2((y_1 - p_{66})/p_{68})))$



Takens embedding theorem: all information required to constrain the model is contained in the observation of one state variable – the membrane voltage V(t) – over a finite time window of duration T.

Record time series data to be fitted: $V(t_1)$, $V(t_2)$, $V(t_3)$, ... $V(t_N)$ induced by the current protocol injected in the neuron $l(t_1)$, $l(t_2)$, $l(t_3)$, ... $l(t_N)$.

Define a cost function to minimize:

$$C(y_1(0), \vec{p}) = \frac{1}{2} \sum_{i=0}^{N} \left\{ V(t_i) - y_1(t_i, \vec{p}) \right\}^2 + u(t_i)^2$$

Observed voltage

Convergence Model variable

Creveling et al, Phys. Lett. A, acceleration function **372**, 2640 (2008)

under:

- Equality constraints: the 12 differential equations linearized at each point of the discretized time window t = iT/N. Typically N=100,000 mesh points \Rightarrow 1.2 million constraints
- **Inequality constraints:** the search intervals of the 72 parameters

Build the Lagrangian by replacing the inequality constraints with logarithmic barriers

 \Rightarrow Karush-Kuhn-Tucker system: sparse systems of linear equations

Interior point optimization: iteratively seek the extremum of the KKT Lagrangian using Newton's method. At each iteration reduce the height of the logarithmic barrier until convergence is achieved.

When data assimilation has converged ~ 2 days of run time on a workstation:

• The $p = \{C_m, g_{Na}, E_{Na}, g_K, E_K ...\}$ vector solution of the problem is obtained – 71 parameters.

Example of extracted parameter sets



naram	naram		lower	upper	neuron	neuron	neuron	param.	param.		lower	upper	neuron	neuron	neuron
number	'name'	units	bound	bound	N1	N2	N3	number	'name'	units	bound	bound	N1	N2	N3
<i>D</i> 1	Cm ¹	μ F/cm ²	0.900	1.100	1.100	1.032	1.035	nameer n20	$K2 h^{5} \cdot V_{1/2}$	mV	-90.000	-35,000	-66.425	-52 586	-73 600
p_2	$NaT:g^2$	nS/cm ²	5.000	170.000	7.545	85.364	9.736	P 39	$K_{2,n} = V_{1/2}$	mV	30,000	5.000	-00.423	11 517	24.863
p_3	NaP:g	nS/cm ²	0.000	20.000	0.008	0.086	0.075	P40	$K_{2,n.K}$	mV	-39.000	5.000	-30.417	20.380	-24.803
p_4	E_{Na}	mV	45.000	55.000	55.000	55.000	55.000	<i>P</i> 41	K2,//.O	III V	-39.000	-3.000	-39.000	-20.369	-3.000
<i>p</i> 5	K1:g	nS/cm ²	0.000	80.000	0.096	0.216	0.000	<i>P</i> 42	$K_{2}, n: \tau_{0}$	ms	0.020	2.000	2.000	1.989	0.020
p_6	K2:g	nS/cm ²	0.000	80.000	5.687	14.708	1.074	<i>p</i> 43	K2,h: τ_{max}	ms	0.500	100.000	68.490	100.000	0.663
<i>p</i> 7	K3:g	nS/cm ²	0.000	12.000	0.438	0.191	6.482	<i>p</i> 44	K2, h : δ^2	ms	0.000	30.000	2.113	0.000	22.460
p_8	E_K	mV	-85.000	-70.000	-75.001	-85.000	-85.000	p_{45}	K3, $m:V_{1/2}$	mV	-15.000	40.000	-15.000	-15.000	5.976
<i>p</i> 9	Leak:g	nS/cm ²	0.010	0.600	0.010	0.036	0.047	p_{46}	K3, <i>m</i> :κ	mV	5.000	65.000	65.000	65.000	37.251
<i>p</i> ₁₀	E_{Leak}	mV	-65.000	-48.000	-65.000	-65.000	-65.000	P47	K3, <i>m</i> : σ	mV	5.000	70.000	70.000	20.202	6.190
<i>p</i> ₁₁	[Ca] _{ext}	mM	0.010	9.000	0.020	9.000	8.996	p_{48}	K3, m : τ_0	ms	0.020	55.000	55.000	0.020	1.106
<i>p</i> 12	HCN:g	nS/cm ²	0.000	10.000	0.017	0.011	0.000	p_{49}	K3,m: τ_{max}	ms	1.000	150.000	150.000	128.333	2.561
<i>p</i> ₁₃	I _{SA} ³	cm ²	15.00	250.0	38.0	81.0	78.0	D50	$CaT.m:V_{1/2}$	mV	-56.000	-8.000	-55,999	-49.049	-55.998
<i>p</i> ₁₄	NaT, $m^4:V_{1/2}$	mV	-45.000	-15.000	-34.738	-18.495	-38.400	P 50	$CaTm\kappa$	mV	5,000	49,000	48,995	29.827	48,991
<i>P</i> 15	NaT, <i>m</i> :κ	mV	0.500	25.000	21.682	22.457	17.327	P 51 D50	$CaTm\sigma$	mV	5 000	55,000	54 990	54 855	54 909
<i>p</i> 16	NaT, $m:\sigma$	mV	0.500	25.000	0.500	0.500	0.500	P 52	$CaT,m:\sigma_0$	me	0.020	2 000	1.066	0.020	1 863
<i>p</i> 17	NaT, $m: \tau_0$	ms	0.010	0.700	0.010	0.194	0.010	<i>P</i> 53	$CaT, m: \tau$	1115	1.000	2.000	204.017	6 272	204 825
<i>P</i> ₁₈	Na I, <i>m</i> : τ_{max}	ms	0.012	7.000	0.012	0.158	0.012	<i>P</i> 54	$Cal, m. t_{max}$	1118 	1.000	295.000	294.917	0.275	294.033
<i>p</i> 19	Na 1, $h:V_{1/2}$	m v V	-/5.000	-35.000	-43.132	-38.452	-57.314	p_{55}	$CaL, m: V_{1/2}$	mv	-80.000	-35.000	-/1.954	-44.457	-/1.213
p_{20}	Na I, $h:\kappa$	m v mV	-25.000	-0.500	-9.400	-4.487	-25.000	p_{56}	$CaL,m:\kappa$	mV	5.000	39.000	38.987	30.094	20.336
P21	NaT $h: \sigma_0$	III V ms	0.020	2000	0.401	0.207	0.841	P57	CaL, $m:\sigma$	mV	5.000	57.000	56.983	22.726	57.000
P22	NaT, $h:\tau_0$	ms	1.000	2.000	30,000	0.207 4.427	21 173	<i>P</i> 58	$CaL,m:\tau_0$	ms	0.020	2.000	1.975	0.020	2.000
P23	NaPm:V _{max}	mV	-69.000	-29,000	-64 537	-37 719	-57 822	p_{59}	CaL, $m:\tau_{max}$	ms	1.000	150.000	149.945	110.617	19.396
P 24 D 25	NaPm: κ	mV	5,000	25.000	5 000	5 349	11 042	p_{60}	$CaL, h: V_{1/2}$	mV	-90.000	-55.000	-72.996	-56.002	-55.000
P 25 D26	NaP $m: \sigma$	mV	5.000	25.000	25.000	5.194	25.000	<i>P</i> 61	$CaL,h:\kappa$	mV	-34.000	-5.000	-33.759	-7.789	-34.000
P 20 D 27	NaP.m: τ_0	ms	0.020	2.000	2.000	0.020	1.088	D62	CaL. <i>h</i> : σ_+	mV	3.000	55.000	54.853	55.000	55.000
P 27 P 28	NaP,m: τ_{max}	ms	0.012	7.000	7.000	7.000	0.012	D63	CaL. <i>h</i> : σ_{-}	ms	3.000	55.000	54.824	4.362	55.000
P 20	$K1,m:V_{1/2}$	mV	-69.000	-21.000	-67.970	-54.343	-67.597	P 05	$CaLh:\tau_0$	ms	5,000	190,000	189,147	59,808	190,000
p_{30}	K1, <i>m</i> : <i>κ</i>	mV	5.000	25.000	6.734	15.528	24.998	P 04	Cal $h \tau_{mm}$	ms	0.500	7000.000	6050.010	7000.000	7000.000
P31	K1, $m:\sigma$	mV	5.000	25.000	7.908	5.000	24.992	P65	HCN h:V.	mV	-00.000	-40.000	-80.061	-63 231	-67 150
P32	K1, m : τ_0	ms	0.020	2.000	2.000	0.052	1.998	P66	$HCN, n.v_{1/2}$	mV	40.000	-40.000	-09.001	-03.231 5.000	-07.150
<i>p</i> ₃₃	K1,m: τ_{max}	ms	1.000	30.000	30.000	1.000	29.979	p_{67}	HCN, <i>n</i> :K	III V	-40.000	-5.000	-0.410	-5.000	-38.130
<i>p</i> ₃₄	$K2,m:V_{1/2}$	mV	-90.000	-21.000	-52.893	-41.932	-66.958	p_{68}	$HCN,h:\sigma$	тv	5.000	40.000	13.075	40.000	38.297
<i>p</i> ₃₅	K2, <i>m</i> : <i>\u03c6</i>	mV	5.000	48.000	13.597	10.972	19.837	<i>P</i> 69	HCN, $h:\tau_0$	ms	0.020	2.000	2.000	2.000	1.029
<i>P</i> 36	K2, <i>m</i> : σ	mV	5.000	48.000	11.528	48.000	45.659	P 70	HCN, $h:\tau_{max}$	ms	100.000	2000.000	2000.000	2000.000	1925.350
<i>p</i> 37	K2, m : τ_0	ms	0.020	2.000	0.020	0.020	2.000	<i>P</i> 71	CaL:p	µm/s	0.000	10.000	0.000	0.010	0.000
<i>p</i> ₃₈	K2,m: τ_{max}	ms	1.000	30.000	1.838	1.000	30.000	<i>p</i> 72	CaT:p	µm/s	0.000	10.000	0.000	0.003	0.004

Assimilation and prediction of neuron output - N1





Assimilation and prediction of neuron output – N2





Prediction neuron response to arbitrary current stimulation





Prediction of the state of ionic gates







Predicting the state of the gate variables of the 9 ion channels – **not accessible to the experiment!**



Separate the experimental signal $V_{data}(t)$ into the useful signal $V_{use}(t)$ and the noise component $v_n(t)$.

Insert in the cost function:

$$\begin{split} c(\vec{x}(0), \vec{p}) &= \frac{1}{2} \sum_{i=0}^{i=N} \left(V_{data}(t_i) - V(t_i, \vec{x}(0), \vec{p}) \right)^2 \\ &= \frac{1}{2} \sum_{i=0}^{i=N} \left(V_{use}(t_i) + v_n(t_i) - V(t_i, \vec{x}(0), \vec{p}) \right)^2 \\ &= \frac{1}{2} \sum_{i=0}^{i=N} \left(V_{use}(t_i) - V(t_i, \vec{x}(0), \vec{p}) \right)^2 + \frac{1}{2} \sum_{i=0}^{i=N} \left(v_n(t_i) \right)^2 \end{split}$$

The cross term cancels as it is proportional to the noise average which is zero. The second term on the RHS gives the variance of the membrane voltage which when driven by thermal fluctuations is given by Nyquist theorem:



Hence:



Provided the noise temperature T is not too large that the second order Taylor expansion is accurate:

$$\delta c = \frac{1}{2} (\boldsymbol{p} - \boldsymbol{p}^*)^T \hat{\boldsymbol{H}} (\boldsymbol{p} - \boldsymbol{p}^*)$$

the true global minimum p^* may be estimated taking a statistical average $\langle P \rangle$ of the multivalued solutions of the data assimilation



Physiologically meaningful parameter solutions

Nogaret et al, Sci.Rep. 6,32749 (2016)

Uncertainty on parameters





Similarity of the activation curves of 2 HVC neurons reconstructed from the inferred global minimum.

DA + Bayesian inference consistently achieves single valued, biologically plausible models.

The method is fully automatic, requiring minimal biological intuition.

It absorbs intrinsic fluctuations of biological neurons to make predictions with sufficient accuracy to evidence trial-to-trial fluctuations in neuron behaviour.



Healthy Heart

Heart Failure

AIR



Loss of Heart rate variability is a prognosis of heart-failure





DA can infer the network connectivity that restores accurate adaptation to physiological feedback

Application of DA: machine learning





3N Central Pattern Generator



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Experiment

Theory



Wojcik et al., Phys. Rev. E 83, 056209 (2011)

4-6 neuron CPGs – electrical switching of motor patterns





0.8

0.6

0.4

 $\Delta \phi_{\rm 21}$

0.2

0

1 0.8 0.6

0.4

 $\Delta \phi_{\rm 31}$

02

0

0

(N-1)! = 120 Attractors



Stimulus induced switching between attractors Demonstration of *command neuron action* to switch motor patterns e.g. gaits

- Scaling of attractors ~(N-1)!
- Switching between attractors is induced by command pulses
- Analog CPGs integrate instantly

DA may "engineer" basins of attraction to produce specific gaits.

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Covariance matrix of the data misfit Hessian







The eigenvalues of the Hessian matrix **H** are the half-axes of the 71D-ellipsoid:

The spectrum of eigenvalue decays exponentially.

The "sloppiest" – most loosely constrained – are the time constants of the model, the most tightly constrained are the gate voltage thresholds.